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Wave Radiation by a Submerged Elliptical Disk

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ABSTRACT

The radiation of small-amplitude waves by an oscillating, horizontally submerged disk of elliptic cross-section located at a finite depth beneath the free-surface is investigated analytically. The theoretical formulation leads to solutions for the fluid velocity potential in terms of a series of Mathieu and modified Mathieu functions of real argument. Numerical results are presented for the added-mass and radiation damping coefficients of the disk in the various oscillation modes for a range of wave and structural parameters.

INTRODUCTION

Recently, Zhang and Williams (1995) presented a theoretical solution to the problem of wave scattering by a disk of elliptic cross-section horizontally submerged at a finite depth beneath the free surface, using an eigenfunction expansion approach. They presented numerical results for the wave-induced force and moments and the water surface elevation in the vicinity of the disk for a range of wave and geometric parameters. Their analysis clearly showed the effect of wave focusing around the rear of the disk. In the present paper, a similar analytical approach will be utilized to study the corresponding problem of the radiation of small-amplitude waves by an oscillating, horizontally submerged disk of elliptic cross-section. The elliptic geometry allows consideration of the effects of wave direction and aspect ratio on the wave scattering characteristics of the body. The scattering and radiation of water waves by a bottom-mounted, surface-piercing elliptic cylinder has been studied by Chen and Mei (1971). In carrying out a separation of variables solution of the governing Laplace equation in elliptic cylindrical coordinates, they obtained expressions for the fluid velocity potentials in terms of an infinite series of Mathieu functions. Later Williams (1985a) presented two approximate solutions to the scattering problem, one based on an expansion of the exact solution of Chen and Mei for small values of the elliptic eccentricity, the other based on an integral equation technique involving the application of Green's theorem. Both methods showed excellent agreement with the exact solution and a considerable saving in computational effort was reported.

Chen and Mei (1973) also investigated the hydrodynamic loading on a stationary platform of elliptical shape partially immersed in the free surface. The formulation was based on linearized shallow water wave theory utilizing a depth-averaged velocity potential. The fluid domain was divided into two regions: one beneath the structure, the so-called internal region, and the other external to the structure extending to infinity in the horizontal plane. Expressions for the velocity potentials satisfying the boundary conditions in each region were obtained in terms of an infinite series of Mathieu functions involving unknown coefficients. These coefficients were determined by matching continuity of velocity and mass flux at the interface between the two regions. The analysis was subsequently extended by Williams (1985b) to include the case of a submerged elliptical structure resting on the seabed; the total and differential scattering cross-sections for both the floating and submerged structures were also presented.

Williams and Darwiche (1988) later reinvestigated the problem of wave scattering by floating and submerged cylinders of elliptic cross-section, this time without the shallow-water (long wave) restriction. The fluid domain was divided into two regions: one above or beneath the structure, the so-called internal region, and the other external to the structure extending to infinity in the horizontal plane. Expressions for the velocity potentials satisfying the boundary conditions in each region were obtained in terms of an infinite series of Mathieu functions involving unknown coefficients. These coefficients were determined by imposing continuity of velocity and mass flux at the interface between the two regions. Because, in this case, the velocity potentials are three-dimensional, both the vertical and the angular coordinates must be considered in the matching procedure. The use of the depth-averaged velocity potentials in previous work effectively eliminated the vertical coordinate from the analysis. Subsequently, Williams and Darwiche (1990) analyzed the corresponding wave radiation problems by a similar technique.

The eigenfunction expansion approach is also adopted in the present problem. The fluid domain is divided into three regions: one above and one beneath the disk, the so-called internal regions; and an external region extending to infinity in the horizontal plane. Expressions for the velocity potentials satisfying the boundary conditions in each region are obtained in terms of an infinite series of Mathieu functions involving unknown coefficients. Again, these coefficients are determined by imposing continuity of velocity and mass flux at the interfaces among the three regions. Numerical results will be presented for the added-mass and radiation damping coefficients of the disk associated with the various modes of oscillation for a range of wave and structural parameters.

FORMULATION

The problem geometry is shown in Fig. 1. Elliptic cylindrical coordinates (u, v, z) are employed, u = constant and v = constant being orthogonally intersecting families of confocal ellipses and hyperbolae, respectively. The z-axis is measured vertically upwards from an origin in the mean free-surface. The transformation from Cartesian to elliptical coordinates is:

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