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Strength and Deformation Capacity of Bends in Pipelines

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ABSTRACT

Bending moments on bends in pipelines may cause severe ovalization and stresses, both in longitudinal and circumferential directions. The bending stiffness depends not only on the radius of the bend, the diameter-to-wall-thickness ratio and the internal or external pressure, but also on the length of the bend, the loads due to the surrounding soil and the bending direction. After start of yielding, the bending moment may still increase considerably. For example, it can be shown that for bends with a radius-to-diameter ratio equal to 3, the maximum moment may be 2 or 3 times greater than the moment where first yielding started. Also the rotation capacity (the ability to follow imposed deformation) is many times greater than elastic theory can predict. In this paper, the structural behaviour of bends is analyzed. Improved analytical models for elastic design and new analytical models for plastic design are presented. The results of the analytical models are compared with test results and the results of finite element calculations.

NOMENCLATURE

$a_2, a_4, \ldots$: Coefficients for in-plane bending of smooth bends
C_o, C_p : Function of λ, and of λ and ψ respectively
D_u : External diameter
D : Average diameter Æ $D = D_u - t$
D_o, D_p : Function of λ, and of λ and ψ, respectively
ΔD_v : Change in vertical diameter (in plane of bend)
ΔD_h : Change in horizontal diameter (perpendicular to plane of bend)
E : Modulus of elasticity
EI_{bo} : Flexural stiffness of a bend
$f_1, f_2, ..$: Coefficients in analysis of bends
G_o, G_p : Function of λ, and of λ and ψ, respectively
K : Curvature due to bending
k_o, k_p : Flexural stiffness factor for pressure $P = 0$ and for P, respectively
k'_o, k'_p : Reduced value of k_o, k_p, respectively
M_m : Maximum moment if longitudinal yielding is governing mechanism
M_{pbo} : Maximum moment if occurrence of plastic hinges in circumferential direction is governing mechanism
M_o : Moment which develops when, on applying Q, angular rotation is kept 0
M_p : Plastic moment $\rightarrow M_p = 4r^2 t\sigma_e$
M_{e1}, M_{e2} : Moment at which value of m_e is first, respectively secondly, attained anywhere in circumferential direction of wall of pipe
m_e : Plate moment per unit width at end of elastic region $\rightarrow m_e = t^2\sigma_e/6$
m_p : Plastic plate moment per unit width $\rightarrow m_p = t^2\sigma_e/4$
m_x, m_y : Plate moment per unit width in longitudinal and circumferential directions, respectively
n_p : Plastic plate normal force per unit width $\rightarrow n_p = t\sigma_e$
n_x, n_y : Plate normal force per unit width in longitudinal and circumferential directions, respectively
P : Difference in pressure between inside and outside of pipeline $P = P_i - P_u$
Q : Earth pressure
Q_d : Directly transmitted earth pressure
Q_i : Indirectly transmitted earth pressure (support reaction)
Q_{eq} : Equivalent earth pressure to transform Q_i to a quantity Q_d which gives same average plate moments in circumferential direction as Q_i does
R : Radius of nonloaded bend
R' : Radius of bend including additional curvature due to bending
R'' : Radius of bend including additional curvature and increased by r for negative bending or decreased by r for positive bending
r : Average (or mean) radius $\rightarrow r = D/2$
t : Pipe wall thickness
t_r, t_b : Pipe wall thickness in straight pipe and bend, respectively
u, w : Displacement of an element of pipe circumference along circumference and perpendicular to circumference, respectively; also $w = \Delta D/2$
σK_{tot} : Total increase in curvature in deformation step concerned
σK_o : Increase in curvature, due to ovalization
σK_v : Increase in curvature, due to longitudinal yielding
σw_v : Increase in ovalization, due to longitudinal yielding
α, β, γ : Loading angle, bearing angle for Q_d and for Q_i and Q_{eq}, respectively
α : Angle of unloaded bend
$\Delta\alpha$: Change of α
η : Relative change of $\alpha \rightarrow \eta = \Delta\alpha/\alpha$
η_{e1}, η_{e2} : η associated with M_{e1} and M_{e2}, respectively
λ : Bend characteristic $\rightarrow \lambda = tR/r^2$
ν : Poisson's ratio
ψ : Bend characteristic $\rightarrow \psi = PR^2/Ert$
ψ_m : Slope of yield surface

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KEY WORDS: Pipeline, pipe, bend, smooth bend, plastic design, deformation capacity.

INTRODUCTION

Pipelines, offshore and onshore, are subjected to combinations of various loadings, such as internal or external pressure, surrounding soil, bending, normal force, shear force and sometimes