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The Instability of Surface Shear Layer Induced by Wind

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ABSTRACT

By the numerical solution of the Rayleigh equation for complex frequency and complex wave number, the stability of four families of surface shear flow, which represent velocity profiles induced by wind over a water surface, has been investigated. It is shown that for flow of Froude number from 0.85 to 1.41, the unstable waves are primarily convective in nature.

INTRODUCTION

A gust of wind blowing over a body of water induces a shear (vorticity) layer in the water just below the surface. This layer has been found to be unstable and may be a source of wind-induced surface waves. This mechanism of generation of surface waves by wind was first considered by Stern and Adam (1973) and further investigated by Voronovich et al. (1980) and Caponi et al. (1991). In particular, these authors studied the stability of wind-induced surface shear layers, which are represented as a layer of constant vorticity of finite thickness Δ and surface velocity u_s, lying above a semi-infinite domain of stagnant fluid. Their investigations show that there exist unstable waves for a range of parameters, i.e., the wave speed c has a non-zero imaginary part corresponding to temporally unstable wave modes for a range of u_s and Δ. The unstable wave modes are in the capillary-gravity regime with wavelengths of the order of centimeters. Caponi et al. showed that a necessary condition for the existence of unstable wave mode is that the surface velocity $u_s > c_m$. Here $c_m = (4\bar{g}\bar{T}/\bar{\rho})^{1/4}$ is the minimum capillary-gravity wave speed over a stagnant fluid layer, where $\bar{g}$ is the acceleration due to gravity, $\bar{T}$ is the surface tension and $\bar{\rho}$ is the associated fluid density. Unstable modes exist when the vorticity layer thickness Δ exceeds a certain critical value Δ_{crit}, which is dependent on u_s.

Morland, Saffman and Yuen (1991) extended the work on temporal stability to cover cases of smooth velocity profiles. With the smooth velocity profiles, Morland et al. obtained the same conclusion as Caponi et al., i.e., instability can only occur if $u_s > c_m$, and for each value of $u_s > c_m$ there is a critical value of vorticity thickness Δ_{crit} such that instability can only occur when $\Delta > \Delta_{\text{crit}}$. However, with a smooth velocity profile, the temporal instability growth rates were found to be significantly smaller than those associated with a piecewise-linear velocity profile.

The temporal stability of a shear layer at a water surface was again studied by Shrira (1993). He introduced a new analytical technique based on a series expansion in terms of a parameter $\varepsilon \sim U''/\omega k$. ε characterises the smallness of deviation of the wave motion from that of the potential wave motion. The series' absolute convergence is guaranteed for $|\varepsilon| < 1$, which limits the general application of the technique. An important feature of the technique is the availability of a priori bound for truncation error.

In this work, we formulate the stability problem as a matrix eigenvalue problem in complex frequency ω. This formulation has an advantage over the shooting procedure used by Morland et al. in that no pre-knowledge of the eigenvalue (guess values) is required. The matrix eigenvalue formulation also reduces the risk that unstable eigenvalues may be missed. Below, we present the temporal stability results for the velocity profiles studied by Morland et al. The results confirm their basic findings. However, temporal stability is a highly idealized form of instability in which the perturbation is assumed to have a regular periodic spatial form. In reality, perturbation in unbounded physical system may be localized. The growth of perturbations which originate from localized disturbance is best described in terms of the spatial-temporal concepts of *absolute instability* and *convective instability*. The classification into absolute and convective instabilities is derived from a time-asymptotic analysis of the spatial-temporal evolution (initial-value problem) of localized disturbances by Bers and Briggs (1964), originally developed for the study of plasma instabilities. A study of the absolute and convective characteristics of piecewise-linear and smooth surface shear layers is reported below.

MATHEMATICAL FORMULATION

Flow Stability

We choose the Cartesian coordinate frame for the problem as follows: the positive x-axis pointing from left to right and lying along the initially undisturbed water surface of the water; the positive y-axis pointing vertically upwards. The fluid is assumed incompressible and inviscid. We denote the two-dimensional parallel primary/basic velocity field by $(U(y),0)$. A small-amplitude normal mode perturbation is added to the basic mean flow $U(y)$, which has the form $(u,v) = (\partial/\partial y, -\partial/\partial x)\phi(y)\exp^{i(kx-\omega t)}$. Here ω and k represent the complex frequency and complex wavenumber of the perturbation, respectively. The disturbance function $\phi(y)$ obeys the well-known Rayleigh's equation, i.e.:

$$(kU - \omega)(\phi'' - k^2\phi) - kU''\phi = 0, \qquad y < 0 \qquad (1)$$

Superscript prime $'$ denotes ordinary derivative taken with respect to y. The flow disturbance is assumed to decay to zero as $y \to -\infty$. Hence:

$$\phi(y) \to 0 \qquad \text{as} \qquad y \to -\infty \qquad (2)$$

Let $\eta(x,t) = \hat{\eta}e^{i(kx-\omega t)}$ denote the displacement of the free surface

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