

## A Nonlinear Oscillator Model for Vortex Shedding from a Forced Cylinder. Part 2: Shear Flow and Axial Diffusion

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### ABSTRACT

A diffusive van der Pol equation with parametric forcing is introduced as a model for vortex shedding from a forced cylinder in a shear flow. The equation is numerically integrated for particular cases of linearly sheared flows. The numerical results show the familiar cellular shedding pattern associated with sheared flows. The value of the turbulent kinematic viscosity identified with the diffusive term is ascertained by matching numerically determined cellular shedding patterns with experimentally determined cellular shedding patterns found by several investigators. Finally, a relationship is postulated between the turbulent kinematic viscosity and the shear parameter for the flow.

### INTRODUCTION

During the past several years, investigators have demonstrated that the Ginzburg-Landau equation (Albarède and Monkewitz, 1992) or its close relative, the diffusive van der Pol equation (Noack et al., 1991), arises as the leading order approximation for the vortex shedding instability from a stationary cylinder in a shear flow. In this paper, we extend the diffusive van der Pol equation to the description of the vortex shedding instability from a forced cylinder in a shear flow. The model equation is given by:

$$\frac{\partial^2 q}{\partial t^2} + \varepsilon \omega_s \left( \frac{4q^2}{Q_0^2} - 1 \right) \frac{\partial q}{\partial t} + \omega_s^2 q - \nu \frac{\partial^3 q}{\partial z^2 \partial t} = \omega_s^2 q F \left( Y, \frac{\partial Y}{\partial t} \right) \quad (1)$$

where  $t$  is time and  $z$  is the axial coordinate along the cylinder. Also,  $q$  is any near wake fluid quantity associated with the vortex shedding instability,  $Q_0$  is the amplitude of  $q$  for uniform flow over an unforced cylinder,  $\omega_s(z)$  is the local shedding frequency,  $\varepsilon$  is a scaling constant, and  $\nu$  is the turbulent kinematic viscosity. The cylinder displacement normal to the flow is denoted by  $Y$ .

The parametric coupling function,  $F(Y, \partial Y/\partial t)$ , was developed by Skop and Balasubramanian (1995) so that, for uniform flow, the boundaries of the fundamental synchronization region as obtained from Eq. 1 coincide with the boundaries of the fundamental synchronization region as determined experimentally by Williamson and Roshko (1988). They (Skop and Balasubramanian) find:

$$F \left( Y, \frac{\partial Y}{\partial t} \right) = A_1 \left[ \text{sign} \left( Y \frac{\partial Y}{\partial t} \right) \right] \left| \frac{Y}{\omega_s D^2} \frac{\partial Y}{\partial t} \right|^\gamma + A_2 \left| \frac{1}{\omega_s D} \frac{\partial Y}{\partial t} \right|^{2\gamma} \quad (2)$$

Here,  $D$  is the diameter of the cylinder and the parameters  $A_1$ ,  $A_2$  and  $\gamma$  are given by  $A_1 = 2.45$ ,  $A_2 = 0.72$  and  $\gamma = 0.34$ . Skop and Balasubramanian also develop the scaling constant  $\varepsilon$  as  $\varepsilon = 0.2$  so

that the response magnification factor for  $q$  as obtained from Eq. 1 corresponds with the lift amplification factor as determined experimentally by Bishop and Hassan (1964).

In this paper, Eq. 1 is numerically integrated for particular cases of linearly sheared flows. The results demonstrate the cellular shedding pattern associated with such flows and the coalescence of the cells under forced vibrations. The turbulent kinematic viscosity is developed by matching numerically determined cellular shedding patterns with experimentally determined cellular shedding patterns found, under different flow conditions, by several investigators (Stansby, 1976; Peltzer and Rooney, 1981; Woo et al., 1981; Peltzer, 1982). To conclude, a relationship is postulated between  $\nu$  and the shear parameter  $\beta$  defined by:

$$\beta = \frac{D_{ref}}{\omega_{s,max}} \frac{\partial \omega_s}{\partial z} \quad (3)$$

where  $\omega_{s,max}$  is the maximum shedding frequency along the cylinder and where  $D_{ref}$  is the diameter of the cylinder at the location of  $\omega_{s,max}$  (in our case,  $D_{ref} = D$ ).

### DISCRETIZATION AND NUMERICAL INTEGRATION

We introduce the dimensionless time  $\tau = \omega_{s,max} t$ , the dimensionless axial coordinate  $\zeta = z/D$ , the dimensionless near wake fluid quantity  $u = 2q/Q_0$ , the dimensionless turbulent kinematic viscosity  $p^* = \nu/(D^2 \omega_{s,max})$ , and the dimensionless cylinder displacement  $y = Y/D$ . In terms of these dimensionless quantities, Eq. 1 becomes:

$$\frac{\partial^2 u}{\partial \tau^2} + \varepsilon \Omega_s (u^2 - 1) \frac{\partial u}{\partial \tau} + \Omega_s^2 u - p^* \frac{\partial^3 u}{\partial \zeta^2 \partial \tau} = u G \left( y, \frac{\partial y}{\partial \tau} \right) \quad (4)$$

Here,  $\Omega_s(\zeta) = \omega_s/\omega_{s,max}$  and the function  $G(y, \partial y/\partial \tau)$  is, from Eqs. 1 and 2, defined by:

$$G \left( y, \frac{\partial y}{\partial \tau} \right) = \Omega_s^{2-\gamma} A_1 \left[ \text{sign} \left( y \frac{\partial y}{\partial \tau} \right) \right] \left| y \frac{\partial y}{\partial \tau} \right|^\gamma + \Omega_s^{2(1-\gamma)} A_2 \left| \frac{\partial y}{\partial \tau} \right| \quad (5)$$

The boundary conditions on Eq. 4 can be chosen from among constrained flow given by:

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