

A Flexible Vertical Sheet in Waves

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ABSTRACT

The problem of a vertical elastic sheet subjected to a train of ocean waves in infinitely deep water is solved. The sheet is assumed to be thin and governed by the Bernoulli-Euler equations, and to extend from the water surface to a finite depth. The solution utilizes the Green's function for water waves and the transformation of the Bernoulli-Euler equation to an integral equation. The model predicts that there will be substantial reflection when the sheet is held rigidly at the top end and penetrates the water to a significant fraction of the wavelength. Resonances at which the reflection is reduced, or is zero, also exist for certain values of the stiffness, mass and frequency. A sheet which is free at both ends does not reflect significantly for any values of mass, stiffness or depth of the sheet.

INTRODUCTION

Problems involving the interaction of waves with floating or fixed bodies have obvious implications to the design of offshore structures, ships, etc., and because of this they have received considerable attention. A number of different forcing regimes have been identified depending on the ratio of the wavelength to body dimension. If the body size is considered as being of the same order of magnitude as the wavelength, then the diffraction problem for the interaction of a linearized wave with the body may be solved by the use of a Green's function. Such methods have found wide applicability and have been used extensively — the basic method is outlined in Sarpkaya and Isaacson (1981) — but only in consideration of rigid bodies; the flexural response of the body is not included. In many cases it is clear that the floating or submerged body will be capable of considerable flexure. The problem of a floating elastic or viscoelastic body acted upon by a wave train has been considered in the context of floating breakwaters by Stoker (1957) and Tayler (1986), neither of whom solved the complete problem.

Recently Meylan and Squire (1993, 1994) have revisited the problem in the context of a floating ice sheet. Modelling floating ice as an elastic beam to calculate its response to ocean waves has been the accepted method for over a century. The scheme used by Meylan and Squire involves the standard Green's function approach, combined with rewriting the elastic equation of the ice as a further integral equation. The results obtained show that a floating elastic body will lead to strong resonances at which the transmission is perfect. These resonances correspond to strain maxima.

The fixed vertical barrier in linear waves was first considered by Dean (1945) and Ursell (1947). Dean solved the problem of a vertical barrier extending from the ocean floor to a height below the water surface. Ursell solved the corresponding problem of a vertical barrier extending down from the water surface. In both cases the solutions were analytic, and they obtained expressions

for the absolute value of the reflection and transmission coefficients in terms of modified Bessel functions. Recently the problem of a vertical barrier extending into the water has been considered by Stiassnie et al. (1984) in the context of vortex shedding.

The vertical elastic body has application to breakwaters, wave dampers and the generation of wave energy. It also has relevance to problems of wave propagation through continuous ice sheets where pressure ridges may drive the sea ice down vertically. The effect of these ice keels on wave propagation has not been quantified to date, and this paper may offer some insight into the processes involved.

PROBLEM FORMULATION

We consider a linear deep-water ocean surface gravity wave impinging on a vertical sheet which is assumed to be perfectly elastic. The problem will be totally two-dimensional. We assume that the sheet occupies the region $0 < z < a$ and $x = 0$ and obeys the Bernoulli-Euler equation, and solve for the wave potential, Φ . In the absence of the sheet Φ must satisfy the following boundary value problem, assuming inviscid and irrotational flow:

$$\begin{aligned} \nabla^2 \Phi &= 0, \quad 0 < z < \infty, \\ \frac{\partial \Phi}{\partial z} &= 0, \quad z \rightarrow \infty, \\ -\rho \left(g \frac{\partial \Phi}{\partial z} - \frac{\partial^2 \Phi}{\partial t^2} \right) &= -\frac{\partial P}{\partial t}, \quad z=0 \end{aligned} \quad (1)$$

where ρ is the density of water and g is the acceleration due to gravity. The Bernoulli-Euler equation of motion for the sheet will be:

$$\frac{\partial^2 w}{\partial t^2} + \mu^2 \frac{\partial^4 w}{\partial z^4} = \frac{P}{\rho' h}, \quad x=0, \quad 0 < z < a \quad (2)$$

where w is the displacement of the sheet, ρ' is its density, h is its thickness, and $\mu^2 = Eh^2/12\rho'(1-\nu^2)$ where E is the effective Young's modulus and ν is Poisson's ratio. We know that the pressure as a function of depth is given by the hydrodynamic term

Received January 14, 1994; revised manuscript received by the editors January 24, 1995. The original version (prior to the final revised manuscript) was presented at the Fourth International Offshore and Polar Engineering Conference (ISOPE-94), Osaka, Japan, April 10-15, 1994.

KEY WORDS: Linear waves, vertical elastic sheet, reflection and transmission.