

# A New Method of Modelling Ice Accretion on Objects of Complex Geometry

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## ABSTRACT

**A three-dimensional random walk model has been developed to predict the shape of ice accretions on objects of complex geometry. As an example, the accretions on a hemisphere and on a cylinder are considered. The influence of the model parameters on the shape of the ice accretion has been examined. The range of parameters for which the icicles reach their maximum length has been identified. The model predicts a variety of accretion shapes including finger-like and curtain-like icicles. It has also been established that, to a first approximation, the icicle length has a uniform distribution.**

## INTRODUCTION

When water droplets encounter an object and the atmospheric conditions are sufficiently cold, ice will accrete on it. Continuous models which are based on conservation laws for mass and heat have been used previously to predict the rate of accretion. Two main categories of such models can be distinguished: time-independent (e.g., Lozowski et al., 1983) and time-dependent (e.g., Szilder et al., 1987; Poots and Skelton, 1990). In spite of significant progress in the development of such two-dimensional models, they provide reasonable results only for simple object geometry and when the initial shape does not undergo substantial changes. The most demanding cases occur when the accretion is very wet and has a complex geometry evolving with time. In such circumstances, only three-dimensional models can give realistic predictions.

During wet icing conditions, when there is plenty of available water, a pendant ice formation may be found under the object. As a result of the limitations of continuous models, the formation of such a large accretion below an object has not been simulated successfully. However, numerical models of freshwater icicles (Makkonen, 1988) and marine icicles (Chung and Lozowski, 1990) have been developed. In this paper, instead of using a continuous approach, a random walk model is presented, which can be used to simulate the three-dimensional ice accretion on and under an object of any geometry.

Monte Carlo models have been used in ice accretion research as an alternative to continuous models. In this method, the motion of each drop or of drop ensembles is examined individually. This approach has been applied successfully to predict two-dimensional accretion under riming conditions (e.g., Gates et al., 1988). Recently, the random walk method has been introduced into ice accretion research (Szilder, 1993). This method allows the simulation of axisymmetric icicles as a function of atmospheric conditions (Szilder and Lozowski, 1994a). The random walk approach has also been used to simulate an individual three-dimensional icicle under a horizontal surface (Szilder and Lozowski, 1994b). The main advantage of the random walk approach is that it models realistically and with numerical efficiency the flow of water

along the surface of the accretion.

The objective of this paper is to develop a three-dimensional model which will predict the evolution of the ice accretion on objects of complex geometry. This model not only calculates the accretion on the side exposed to impinging droplets, but also the accretion below the object, including the formation of multiple icicles. The model also allows investigation of the stochastic variability of the accretion shape.

## MODEL DESCRIPTION

The model we will describe is a combination of a ballistic trajectory and a random walk model. The ballistic model determines the location of impact of the fluid elements; the random walk model predicts their motion along the surface of the ice already formed on the object. The fluid element in the model may be imagined to consist of many small drops or to be part of a single large drop. The details of the two-dimensional and three-dimensional versions of the model have been described by Szilder and Lozowski (1994a) and Szilder and Lozowski (1994b), respectively. However, the highlights of the three-dimensional version of the model including the ballistic trajectory component are given below.

The accretion domain is defined by a three-dimensional rectangular lattice. Fluid elements are fired along straight vertical trajectories from a random position above the object. A fluid element impinges onto the existing ice accretion if it reaches a lattice location just above a lattice box already occupied by an ice or substrate element, or if it encounters a location where one of eight neighbouring cells, located on the same horizontal surface, is already occupied. The second condition corresponds to the case in which a fluid element is intercepted by just grazing an already frozen element. From the location of the impact, the fluid element begins its random walk along the ice structure.

The random walk of a fluid element consists of a series of moves through the three-dimensional lattice. At each step there are seven possibilities: the fluid element may move one cell in any of six perpendicular directions, or it may freeze in situ. In the present work, it is assumed that the probabilities of moving horizontally, in four perpendicular directions, and upward, are the same. The ratio of the probability of moving downward to the probability of motion in any other direction is defined as the motion parameter. The value of the motion parameter is likely associated with the relative magnitudes of gravity, surface tension and viscous forces. Based on the numerical experiments of Szilder and Lozowski (1994b), the motion parameter has been chosen to be 3.

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**KEY WORDS:** Freezing process, ice accretion, icicles, 3D icing modelling.