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Third-Order Analysis of Nonlinearities Bounded to Narrow Banded Spectra

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ABSTRACT

A new approach to nonlinearities that arise in random sea wave behaviour is proposed. The problem is resolved at the third order for an irregular narrow-bounded wave train. The solution is obtained by considering the wave behaviour as a system whose characteristic is derived from the classic third-order Stokes solution for regular waves. It is shown how, at the third order, nonlinear interactions do not influence either bounded long wave components, which are found at the second order, or high twice peak frequency components. Instead, new modulated components connected with the linear spectrum are found to arise.

INTRODUCTION

In the last few years, many researchers have devoted great effort to studying some maritime phenomena that cannot be described adequately by linear wave theory, especially in the case of wave trains of finite amplitude.

In the past, by means of deterministic or statistical approaches, several authors have shown that some information, such as wave asymmetry or build-up of bounded waves, cannot be explained by a linear approach. Analytical studies by Phillips (1960), Hasselmann (1962) and Longuet-Higgins and Stewart (1962, 1964) are well-known and considered fundamental by most researchers. In their second-order studies, these authors predicted bounded waves by means of nonlinear theories based on perturbation solutions of Euler equations.

Later, other authors, again using perturbation methods, carefully examined past studies at the second order, applying their results both to analysis of the outcomes of physical model tests (Bowers, 1980; Ottesen-Hansen et al., 1980; Sand, 1982; Barthel et al., 1983) and to digital simulations of nonlinear wave random seas (Hudspeth and Chen, 1979).

Osborne and Boffetta (1989), using the multiple-scale method, have recently shown that second-order bounded waves can also be explained analytically by the nonlinear Schrödinger equation (NLS), proposing a generalised form of the radiation stress first introduced by Longuet-Higgins and Stewart. By means of a system theory approach, Petti (1991) has shown that second-order wave components can also be explained by this technique. The advantage of this method is that linear and nonlinear components can be separated analytically, once the global spectrum is known. In the present framework, by means of the same technique already used by the author to study second-order phenomena, a new approach to nonlinearities arising in irregular sea wave behaviour is proposed. The problem is now approached at the third order for an irregular narrow-bounded wave train.

As pointed out by Borgman (1982), a third-order problem requires knowledge of third-order moments and their frequency domain counterparts (i.e., bispectrum). Following these indications, Elgar and Guza (1986) developed a perturbation theory based on bispectral analyses. However, due to the complexity and large amount of data needed to calculate the third-order components, this technique is not easily applicable.

In this paper, a new analytical third-order method is proposed, according to which wave behaviour is studied as a system whose characteristic is derived from the classic third-order Stokes solution for regular waves. It is shown that, at the third order, nonlinear interactions do not influence bounded second-order long waves, whereas new modulated components arise round the fundamental peak frequency. Moreover, an analytical method to separate both linear and nonlinear autocorrelation and spectral components is derived. Lastly, some results from a series of flume experiments are presented and compared with the proposed theory.

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NONLINEAR SYSTEM ASSOCIATED WITH THIRD-ORDER STOKES SOLUTION

A third-order one-dimensional Stokes regular wave $\eta(x,t)$ traveling along direction x at time t on an impermeable horizontal bottom can be written as:

$$\eta(x,t)=-a^3 f_3(k,h)\cos(kx-\omega t)+\sum_{n=1}^{3} a^n f_n(k,h)\cos[n(kx-\omega t)] \quad (1)$$

k being the wave number and ω the angular frequency; $f_1(k,h)$, $f_2(k,h)$ and $f_3(k,h)$ are functions of parameter k and depth h, equal to:

$$f_1(k,h)=1 \quad (2)$$

$$f_2(k,h)=\frac{k}{4}\left[\frac{3}{\tanh^3(kh)}-\frac{1}{\tanh(kh)}\right] \quad (3)$$

$$f_3(k,h)=\frac{3}{32}k^2\left[4+\frac{8}{\sinh^2(kh)}+\frac{5}{\sinh^4(kh)}+\frac{3}{\sinh^6(kh)}\right] \quad (4)$$

and a equal to $H/2$, H being wave height defined as usual as the vertical distance between wave crest and wave trough. Eq. 1 was obtained by Brink-Kjær (see Svendsen I.A. and Jonsson I.G., 1980) under the hypothesis of wave motion with zero net mass flux.

Parameters ω, k, h and a are related to gravitational acceleration g by means of the dispersion relation: