

# Simulation of Nonlinear Waves and Forces Due to Transient and Steady Motion of Submerged Sphere

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## ABSTRACT

This paper presents a time domain simulation of fully nonlinear waves and nonlinear hydrodynamic forces that are generated by a submerged sphere in a transient and steady motion near the free surface. A higher-order boundary element method previously developed in frequency domain was further developed to solve the boundary integral equation in time domain. The multiple node technique was applied on elements in the neighborhood of the edges and corners of the truncated rectangular numerical wave tank. Orlanski's open boundary condition was modified in association with the boundary element method to satisfy the radiation condition at the upstream and downstream boundary. The initial force variation provides the impacting hydrodynamic force (added mass) and the near-steady forces give the averaged steady nonlinear hydrodynamic forces. The nonlinear stern wave appears to be steep and asymmetric and its progressing speed is faster than that of the linear one.

## INTRODUCTION

This paper deals with a development of a time domain simulation technique for the analysis of fully nonlinear waves and hydrodynamic forces that are induced by translation of a submerged sphere. Because the motion starts from rest in calm water, the study involves the transient and steady motion and effect of these motions on the free surface waves and hydrodynamic loads.

The nonlinear wave-body interaction may be largely divided into three problems: a body moves at a prescribed manner in calm water, in which case the body's motion-induced nonlinear force is of interest; a body is fixed in nonlinear waves, in which case the nonlinear wave load is to be computed; the combination of the foregoing two cases, namely the nonlinear load on a structure moving at a prescribed mode or induced by nonlinear incident waves. In addition to these, the body may be submerged or surface-pierced. The former is simpler than the latter. Therefore we chose, as a preliminary work, the simplest case, i.e., a submerged sphere that is set into motion from rest.

We have two numerical studies applying the linear and fully nonlinear free surface boundary condition. Analysis of the results is specifically centered firstly on the hydrodynamic impact relating to the hydrodynamic added masses that are affected by imposition of the two different free surface conditions; secondly, on the linear and nonlinear wave forces at the near-steady state; and finally, on the linear and nonlinear ship waves.

We recently developed a higher-order boundary element method (HOBEM) for predicting linear and second order wave forces on a large structure in the frequency domain (Liu et al., 1991, 1993). The HOBEM was evaluated and believed to be the most efficient numerical technique today for solving wave loads in the frequency domain (Liu et al., 1991).

In order to apply the HOBEM to the fully nonlinear time domain problem we developed a few more necessary items, i.e.,

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the computation of wave slopes and particle velocity of the free surface, the time step integration procedure with Eulerian description, multiple node technique for the elements in the region of the corners and edges at the upstream and downstream boundaries of the wave tank and the modification of Orlanski's open boundary condition (obc).

There is literature of the time step integration and open boundary conditions. Longuet-Higgins et al. (1976) successfully used the Adams-Bashforth-Moulton time stepping scheme for the simulation of unsteady two-dimensional nonlinear breaking waves. Jogannathan (1985) and Romate (1989) gave general reviews of open boundary conditions. Orlanski's (1976) scheme or its modifications have been widely used with a truncated small numerical wave tank (Chan, 1977; Han et al., 1983; Cheung, 1991; and Yang et al., 1992).

We employ the 4th-order Runge-Kutta (R-K) time stepping integration and Orlanski's obc (1976) with a modification.

## THEORETICAL FORMULATION

Consider a fully submerged body that is horizontally moving with acceleration, starting from rest in the previously undisturbed fluid of an unbounded domain. The acceleration is sustained at a certain mode from rest to a final constant speed  $U_o$ , and the constant speed motion is continued until the simulation ends. The body moves at  $-u(t)$  as shown in Fig. 1, which illustrates a truncated numerical wave tank with assigned symbols indicating the boundaries including the sphere. As usual, we employ the body-fixed moving coordinate system, whose origin is at still water level. The positive  $z$ -axis points upward and the positive  $x$ -axis is directed to the right, which is on the mean water level. All the coordinates and physical variables are nondimensionalized by the characteristic length  $L$  that represents the diameter of the sphere  $D$ , and the characteristic velocity  $U_o$  that represents final velocity. The Froude number is defined by  $F_r = U_o / \sqrt{gL}$ , where  $g$  is gravitational acceleration. Then the nondimensionalized variables are shown below:

$$\begin{aligned}(x', y', z') &= L(x, y, z), \quad \zeta' = L\zeta, \quad (\phi', \Phi') = LU_o(\phi, \Phi), \\ P' &= \rho U_o^2 P, \quad t' = t(L/U_o), \quad U'(t') = U_o u(t)\end{aligned}\quad (1)$$