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Dynamic Instability of a Parametrically Excited Ship Rolling Model

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ABSTRACT

This paper investigates the nonlinear interaction behaviour of the heave-excited rolling of ships. The coupled heave-roll system is shown to have two coupling terms due to parametric excitation: A linear term, usually included in most roll stability analyses, which causes Mathieu's type of instability, and a quadratic term, which is often ignored. The significance of including the quadratic coupling effect of heave is studied. The analytical conditions of heave-excited ship rolling are derived using the harmonic balance method. It is shown that the quadratic coupling term is not a high order term and should be included in the governing equation of motion. The parametrically excited heave-roll behaviour should be modelled by a Hill equation instead of a Mathieu equation.

INTRODUCTION

The dynamic behaviour of ships in extreme waves is highly nonlinear and very complex. In recent literature, several research programs based on experimental investigations of ship capsizing have been reported, e.g., de Kat and Paulling (1989) and Grochowalski (1989). The former also discussed numerical simulations of the large amplitude motions of ships; several modes of capsizing were suggested. One of the most dangerous capsize modes discussed was the parametrically excited large roll response. Even without direct roll excitation, rolling motions can be excited indirectly by heave or pitch of the ship through the coupling effect between these motions. Parametric excitations are dangerous to ship stability because they introduce a time-varying component to the stiffness, or restoring moment, term of the governing dynamic equation. Sudden loss of ship stability can occur when the restoring moment is reduced significantly,

The problem of roll instability induced by heave-roll coupling has been studied by many researchers (Paulling and Rosenberg, 1959; Blocki, 1980; Nayfeh, 1988; de Kat and Paulling, 1989; Tondl and Nabergoj, 1990; Sanchez and Nayfeh, 1990). The rolling motion is usually considered to be governed by a Mathieu-type equation which includes a time-varying periodic stiffness term in addition to the normal system stiffness. The additional term can be attributed to the extra restoring moment of the changing buoyancy caused by the heave motion. A closer look at the problem reveals that the changing buoyancy, together with its changing moment lever, can introduce another parametric excitation term to the system stiffness (Blocki, 1980; Liaw et al., 1993). The problem can be more accurately described by a Hill equation. The numerical results of a model based on the Hill equation were presented in a previous paper (Liaw et al., 1993). The present paper investigates analytically the roll instability behaviour based on the Hill equation and compares the derived instability regions to those based on the Mathieu equation,

In order to evaluate the effects of heave-roll coupling on roll instability and compare it to the classical model of the Mathieu equation, the model under consideration includes only the coupling of roll and heave. The influence of pitch, surge, sway and yaw on the roll instability is ignored; three-dimensional phenomena such as broaching and pitch-roll-heave coupling are not considered.

RESTORING ROLL MOMENT AND ROLLING MOTION

If only two-dimensional motions are considered, the coupled heave-roll behaviour of a ship can be idealized as a plane motion represented by a rotation θ (roll) and a vertical displacement Z_T (heave), as indicated in Fig. 1. The free water surface shown is the "mean" water surface along the length of the rectangular ship. The total heave, Z_T, measured at the center of gravity of the ship can be written as:

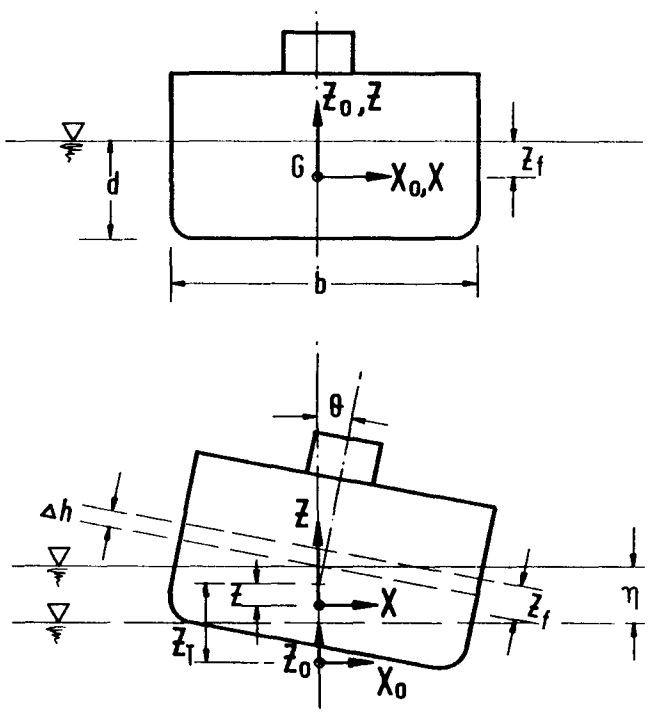

Fig. 1 Coupled heave-roll motion of vessel in following wave; change of draught, Δh, has both linear and quadratic effects on hydrostatic restoring moment

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