

# The Analysis of Two-Level System of Boundary Integral Equations for Fluid-Structure Interaction

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## ABSTRACT

The advanced formulation of the boundary integral equations method is suggested for forced vibrations of an elastic structure immersed in and/or filled by compressible unviscous liquid. A structure is supposed to consist of plates and parts of spherical and cylindrical shells. The first-level boundary equations govern the dynamics of the above-mentioned "simple" parts of a structure. They contain unknown boundary displacements and forces, contact acoustic pressure and driving loads. The kernels of these equations are the well-known Green's functions of "simple" unbounded structures. These functions are presented in analytical form. The second-level boundary equation governs the interaction between liquid and structure. The classical boundary equation related to contact acoustic pressure is modified by substitution of Somigliana's type formulae for normal displacements on each part of a structure. As a result, the second-level equation constitutes the connection among contact acoustic pressure, driving loads and boundary displacements and forces on each part of a structure. The kernels of this boundary equation are convolutions of Green's functions of simple unbounded structures and unbounded liquid.

## INTRODUCTION

Investigations of structural-acoustic coupling problems are mostly based on the methods of numerical analysis. The common approach consists of replacing the exact formulation of the problem by its finite-element approximation. To consider an unbounded acoustic medium, the semi-infinite elements are used which satisfy the Sommerfeld conditions. In this formulation the system of linear algebraic equations contains all unknown amplitudes of displacements along the surface of the structure and the amplitudes of acoustic pressure in the liquid. This system has high order and special attention has to be paid to the storage and handling of this information.

The alternative approach — a combination of finite elements for a structure and boundary elements for an acoustic medium — has appeared to be more efficient. The boundary integral equation related to acoustic pressure is replaced by a system of linear algebraic equations which connects the values of the acoustic pressure at the nodes of the boundary element mesh with normal velocities at the same points. Substitution of the solution for this system to the F.E. approximation of equations for the structure gives traditional F.E. approximate relationship between the amplitudes of the displacements and the driving loads. The matrix of rigidity for the "dry" structure is then modified by the so-called added mass matrix, which is obtained by the inversion of the previously mentioned system of linear algebraic equations for acoustic pressure.

The valuable advantage of this method of numerical analysis is that it permits one to utilize all the well-known F.E. packages (NASTRAN, ANSYS, COSMOS etc.) to solve a structural-acoustics coupling problem. These packages have to be supplemented by a module to generate the added mass matrix. Various assumptions may be used while evaluating the added mass. An incompressible liquid model is used (Bessho, Kawabe, Iwasaki, 1986) to analyze the spatial problem of vibrations of a ship's hull. The D.D.A. (double asymptotic approximation) technique is suggested

by Geers and Felippa, (1978); its accuracy is checked in the particular problem of vibrations of a full spherical shell in a liquid. The well-known NASTRAN package is used along with the new-developed module for evaluating the added mass (Everstine and Henderson, 1990). This package is used for analysis of vibrations of a spherical shell.

One more way to analyze structural-acoustics coupling problems is the Green's functions method. The main idea of this approach is to replace the differential equations of vibrations for the whole structure by an integral equation. This approach has been used by Averbukh and Veitsman (1986) to analyze the simple problem of vibrations of a cylindrical shell of finite length in an infinitely long rigid baffle. The Green's function for the shell was constructed by the Fourier transform method. The finite element method was used (Ross and Lahe, 1990) to construct the Green's function for the plane structure. The boundary integral equations are formulated for both the acoustic medium and the elastic structure (Tanaka and Masuda, 1987), and this system of equations is solved in a manner typical for F.E.-B.E. coupling. The static Green's function was used in the formulation of the boundary integral equation for the structure. A further development of numerical methods in structural-acoustics coupling problems is a formulation of the problem in a way which permits us to consider complex thin-walled structures, but also gives room for detailed asymptotic analysis of each particular problem. These aims may be achieved by the use of the advanced formulation of the B.I.E. method suggested and developed by Slepyan and Sorokin (1989). The main idea of this method is the consistent use of boundary integral equations to describe both the interaction of acoustic medium with the structure and the interaction of elementary parts of the structure with each other.

## A SYSTEM OF BOUNDARY INTEGRAL EQUATIONS

A thin-walled structure considered here consists of  $N$  parts. Surfaces corresponding to each of them are denoted  $S_n$ . Let each element of a complex structure be a fragment of an unbounded cylindrical or spherical shell or plate. Only  $M$  parts of a complex structure are in contact with the acoustic medium. A driving