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# Structural Dynamics of Suspended Cables Supporting Arrays of Offset Bodies Part I: Out-of-Plane Response

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## ABSTRACT

**A general model is derived for the dynamics of suspended cables that support an array of offset bodies, e.g., cable/hydrophone arrays. The general model, which governs geometrically nonlinear, three-dimensional response, is used herein to analyze the linear out-of-plane response of suspensions with small equilibrium curvature. Closed-form solutions are derived for free response (natural frequencies and mode shapes) and for forced response under harmonic end excitation. Solutions for forced response reveal the existence of a "pass-band/stop-band" structure for frequency response. For excitation frequencies within the pass-band, vibration energy freely propagates from the source of excitation throughout the cable/body structure. By contrast, little vibration energy propagates from the excitation source for excitation frequencies in the stop-bands. Studies for free response highlight the key system parameters that have the greatest influence on the natural frequency spectrum.**

## INTRODUCTION

Suspended marine cables are commonly used in applications that require long and easily deployable structural elements. Due to their flexibility, however, suspended cables are easily excited by a variety of sources, which may produce undesirable dynamic response. For example, suspended cables submerged in cross-flows may experience vortex-induced oscillations (Griffin, 1985), commonly referred to as *strumming,* which, in addition to magnifying cable drag forces, may degrade the performance and positioning of instruments attached to the suspension. Wave loading (either directly or indirectly) on the cable can lead to similar degradations of instrument performance and positioning.

The study of cable dynamics has enjoyed a long and rich history, as detailed in Irvine (1981) and recent developments in this field are reviewed in Triantafyllou (1991). A significant understanding of the dynamics of suspended cables has been achieved through a series of analytical studies dating back to the 18th century. Key amongst these is the pioneering study of Irvine and Caughey (1974), which describes the theory of the free vibration of a suspended, elastic cable. This cable theory, which applies to suspensions with small sag, has been extended to describe cables with inclined supports (Triantafyllou, 1984) and single (Cheng and Perkins, 1992a) and multiple (Cheng and Perkins, 1992b) attached masses.

The objective of this study is to investigate the dynamics of a suspended cable that supports an array of offset bodies (e.g., hanging hydrophones) (Fig. 1). As a first step toward this objective, the present study focuses on the linear structural dynamics of the cable/pendulum system alone. The effects of hydrodynamic loading are presently ignored. For the linear response considered herein, added fluid mass and damping terms could be added; see, for instance, Ramberg and Griffin (1977).

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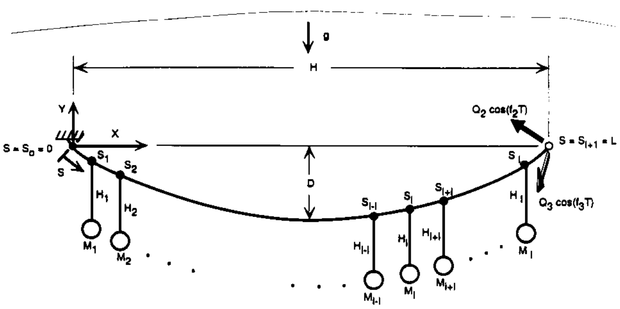

Fig. 1 Definition diagram for cable/body suspension. Horizontally suspended cable supports array of $l$ offset bodies. Each body modeled as pendulum with attachment point $S = S_i$, offset distance $H_i$, and mass $M_i$, $i = 1,2, \dots l$.

In Part I, a general theoretical model is derived that governs the three-dimensional, nonlinear dynamic response of cable/body suspensions. The general model is linearized about a planar equilibrium with small sag. The resulting model for out-of-plane response reduces to that of a taut string supporting an array of pendula and is similar to the string/spring-mass combinations of Iwan and Jones (1984). Solutions for free and forced out-of-plane response are analyzed using closed-form methods. Examples illustrate the pass-band/stop-band structure that governs the propagation of vibration energy along the suspension. The companion model for in-plane response is developed and analyzed in Part II.

## GENERAL MODEL: 3-D RESPONSE

### System Definition

The system of interest is illustrated in Fig. 1 and consists of a horizontally suspended cable supporting an arbitrary array of $l$ offset bodies. In the absence of excitation ($Q_2 = Q_3 = 0$), the suspension sags by an amount $D$ under gravity $\vec{g}$ and forms a planar equilibrium. The equilibrium arc length coordinate $S$ is measured from the stationary left support $S = 0$ to the right support $S = L$. Each body is modeled as a simple pendulum composed of a point mass and a massless, rigid rod that is simply supported at its