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Local Bending Stresses in Cables

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ABSTRACT

After a discussion of the various types of stiffness possible in a suspended cable responding to live loading, theory is presented for the steady-state response of a taut, flat cable in which allowance is made for flexural effects at its terminations. The steady-state response could be taken to refer to strumming effects due to, say, vortex shedding in an underwater application where, if the cable material is synthetic, it will be straight on account of the neutral buoyancy, or nearly such, of synthetic cables. Equally, it could refer to a bridge stay cable. In any event, the model presented for behaviour in the flexural boundary layers of cables provides a useful starting point for the development of fatigue theories for different types of response such as that induced by vortex shedding, or by parametric excitation (where motion at one termination may give rise to lateral motion in the cable). The appendix contains formulae for any type of cable — whether solid, parallel lay or helical lay.

INTRODUCTION

When it is realised that a well-known, large-span suspension bridge, now about 30 years old, has had its complete set of hangers replaced twice (at a cost greater than that of the whole bridge originally), and that this maintenance was necessary because of fatigue damage to the hangers in the immediate vicinity of the sockets, the topic of flexure in cables assumes a special relevance. There is, around the world, growing concern about the serviceability of cables due to a combination of wear and tear and corrosion. Being tensile elements of limited redundancy, the problems of in-service damage to cables — from whatever source — can readily assume profound proportions.

Another area where the profession may have to sort out major technical problems in the future relates to the late-20th century fad of cladding buildings in glass: The long-term behaviour of some of these materials is not well understood.

Suspended cables have different ways of resisting load. The transverse stiffness of a cable of length l, tension T, and flexural rigidity EI is geometric — that is, T/l — and flexural to some degree — that is, EI/l^3. The extent to which flexural stiffness is important can be measured by the size of the independent dimensionless parameter EI/Tl^2. In the vast majority of cable problems this parameter is very small, but this does not mean that it is everywhere unimportant, as we shall see presently.

The axial stiffness of a cable is dependent on two effects: Hookean, or axial in the classical sense, given by EA/l and gravitational, associated with pulling the sag out of a suspended cable, and given by $12T/l(mgl/T)^2$, in which mg is weight per unit length of the cable. The other independent dimensionless parameter of cable mechanics is a ratio of these latter two axial stiffnesses, namely:

$$\frac{1}{12}\left(\frac{mgl}{T}\right)^2\frac{EA}{T} \quad or \quad \frac{\lambda^2}{12}$$

where λ^2 is the name given the parameter by Irvine (1974, 1981, 1992).

A good deal of the author's work over the past 20 years has been devoted to investigations of the curious properties of the parameter λ^2, which is one of the most unusual — and yet fundamental, since it is a ratio of Hookean and gravitational effects — in the whole field of structural mechanics. In the present work it suffices to say that there is an analogy between λ^2 and the Froude Number of free surface flows. The Froude Number is a ratio of inertia forces and gravitational forces. When $Fr > 1$, supercritical flow exists and surface disturbances can pass downstream only. When $Fr < 1$, the flow is subcritical and surface disturbances can pass in either direction. On the other hand, a suspended cable behaves like a taut string when $12/\lambda^2 > 0.5$, and everywhere in the cable deflections of the same sign are generated by the application of a transverse load. However, when $12/\lambda^2 < 0.5$, the suspended cable behaves like a catenary and application of a transverse load causes the cable to seesaw: That is, deflections both positive and negative are generated along the length of the cable.

In the present paper we shall deal only with cables that are essentially taut and flat, that is, $\lambda^2 << 1$ or, more to the present point, $12/\lambda^2 >> 0.5$. We need not consider the effects of gravity any further. In addition, we shall confine our attention to members which are almost completely cable-like. By this we mean, to use a quotation attributed to James Bernoulli which cannot be improved upon, "The action of any part of the line upon its neighbour is purely tangential." Nevertheless, abrupt changes in slope at, say, the positions of concentrated transverse loads which, according to the classical cable theory of the funicular polygon, must occur are in practice much smoother, the smoothing effect being due to the small, but finite, flexural rigidity that all real cable systems possess. In fact, at such locations, behaviour is governed largely by flexure (that is, by a fourth order differential equation) whereas, elsewhere in the span, behaviour is governed by cable action (that is, by a second order differential equation).

This conjures up the concept of the flexural boundary layer — local regions where flexure is important — set in line elements in which cable action governs. Or, from fluid mechanics, flow very near a body is governed by viscosity while, further out in the fluid, the free steam inviscid flow obtains. The mathematics of the two processes — fluid and cable — are identical and, in fact, the suspension bridge was drawn on by Cole (1968) to illustrate

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