International Journal of Offshore and Polar Engineering
Vol. 3, No. 3, September 1993 (ISSN 1053-5381)

Investigation of Large Amplitude Nonlinear Dynamics of Hanging Chains

C.T. Howell
Shell Development Company, Houston, Texas, USA

M.S. Triantafyllou*
Massachusetts Institute of Technology, Cambridge, Massachusetts, USA

ABSTRACT

The three-dimensional nonlinear dynamics of a hanging chain, driven by a large amplitude excitation at the top, are studied numerically and experimentally. Analytic results predict the chain to lose tension over a region adjacent to the free boundary. Numerical results show that large impulse-like tension forces cause displacements which lead to a loss of tension and subsequent collapse of the chain. This low-tension behavior is found to be confined to a small region near the free end, while the remaining chain responds like a taut cable. Incorporating bending stiffness is shown to be an effective numerical tool for enforcing the compatibility relations through computationally difficult periods in which the tension vanishes, allowing simulations to continue through the time at which the chain intersects itself. Experimental results were also obtained which confirm qualitatively and quantitatively the numerical predictions. The chain was found to lose tension, collapse, and eventually revert back and intersect itself experimentally as well.

INTRODUCTION

Highly tensioned chains and cables are used extensively in the offshore industry. Low-tension cables, however, have recently seen increased use largely due to the advent of synthetic cables, for which the weight is much less than that of steel cables of equivalent strength, and thus are close to neutrally buoyant in water. Typical applications for low-tension cables include neutrally buoyant marine cables supporting hydrophones, either drifting freely or moored loosely, and space tethers supporting instrument packages from satellites. In addition, the use of remotely operated vehicle tethers and other cables which involve fiber optic lines for transporting communication signals and power has created systems in which, by necessity, the tension must be kept low.

A different class of problems can be described as low-tension applications as well. These include, for example, long towed arrays which during sharp maneuvers may lose tension entirely, operating as a low-tension cable for periods of time even though the initial tension may be high.

The term low tension is used to refer to problems in which the static tension is significantly less than the dynamic tension. The dynamic behavior differs greatly between low-tension and highly tensioned or taut cables. Low-tension problems are particularly complex because the dynamics cannot be simplified by linearizing the tension, as is typically done for taut cables. In addition, the dynamic tension may act to cancel the static tension over a portion of the loading cycle, thereby giving rise subsequently to impulse-like tension forces. Routh (1860) was the first to study the impulsive loading of cables. More recently, Triantafyllou and Howell (1992a) have studied impulsive cable loadings using asymptotic and numerical techniques.

A chain hanging freely under its own weight exhibits both high-tension (near the top boundary) and low-tension (near the free end) behavior. Therefore, the hanging chain problem affords an opportunity to study the dynamics in both regions as well as the transition between regions. Due to the relatively small restoring force, large amplitude displacements may occur near the lower boundary. Therefore, the effect of geometric nonlinearities becomes more pronounced than with taut cables. By neglecting fluid loads the effect of geometric nonlinearities can be isolated and studied in detail.

Howell and Triantafyllou (1993) obtained asymptotic solutions for the three-dimensional nonlinear hanging chain equations using a perturbation expansion and the method of multiple-time scales. They found the response, due to a harmonic excitation at the top, was sensitive to the excitation frequency and amplitude. In addition, their results reveal that above a certain excitation amplitude, and near resonance, negative tensions occur near the free end. Since negative tensions cannot be sustained, the chain loses tension over a portion of its length.

This paper concentrates on the dynamic nonlinear response to large amplitude excitation. The dynamics leading to the loss of tension, and the subsequent collapse of the chain, are studied through numerical and experimental means.

GOVERNING EQUATIONS

The problem under consideration is a hanging chain in air, as shown in Fig. 1. A lagrangian coordinate system, resolved into tangential, normal and binormal directions $\hat{t}$, $\hat{n}$, and $\hat{b}$, respectively and fixed on the chain, was chosen. The lagrangian coordinate along the chain is denoted by s. The unknown quantities are comprised of the following: tension T; tangential, normal, and binormal velocities u, v, and w, respectively; and two Euler angles ϕ and θ which fix the chain's location in space. The chain mass/unit length is given by m and g denotes the gravitational constant. The governing equations and compatibility relations, as derived by Triantafyllou and Howell (1993a), are given as follows:

---

*ISOPE Member.

Received March 30, 1992: revised manuscript received by the editors June 21, 1993. The original version (prior to the final revised manuscript) was presented at the Second International Offshore and Polar Engineering Conference (ISOPE-92), San Francisco, June 14-19, 1992.
KEY WORDS: Cables, chains, low tension, bending stiffness, nonlinear.