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Combined Creep and Yield Model of Ice Under Multiaxial Stress

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ABSTRACT

A combined creep and yield model has been developed for isotropic ice in a multiaxial stress state. The equations of the model describe the entire creep process, including primary, secondary, and tertiary creep, at both constant stresses and constant strain rates in terms of normalized (dimensionless) time $\bar{t}=t/t_m$. Secondary creep is considered an inflection point defining the time to failure (t_m). The minimum strain rate at failure is described by a modified Norton-Glen power equation, which, as well as the time to failure, includes a parabolic yield criterion. The yield criterion is selected either in the form of an extended von Mises-Drucker-Prager or an extended Mohr-Coulomb rupture model. The criteria take into account that at a certain magnitude of mean normal stresses (σ_{max}) the shear strength of ice reaches a maximum value due to local melting of ice. The model has been verified using test data on yield of polycrystalline ice at –11.8°C and on creep of saline ice at –5°C, both under triaxial compression ($\sigma_2 = \sigma_3$).

INTRODUCTION

The successful solution of ice engineering problems depends greatly upon the accuracy of the constitutive laws and failure criteria of ice used in analyses of engineering structures. Two approaches can be distinguished in describing the deformation and failure processes of ice in uniaxial as well as multiaxial stress states.

In the first (traditional) approach, it is assumed that the total creep shear strain γ^c can be broken down into four components:

$$\gamma^c = \gamma_e + \gamma_p + \gamma_v + \gamma_t \tag{1}$$

where γ_e, γ_p, γ_v and γ_t are elastic, primary, secondary, and tertiary pure shear strain, respectively. In practice, however, to simplify the parameter evaluation procedure and/or to obtain closed-form solutions of boundary problems, some of the strain components in Eq. 1 are often ignored, and the time-dependent creep process in ice is portrayed by simplified models: primary ($\gamma_v = \gamma_t = 0$), secondary ($\gamma_p = \gamma_t = 0$), and tertiary ($\gamma_p = \gamma_v = 0$) creep equations, often represented by various mechanical models. Combined models such as attenuating ($\gamma_t = 0$), exponential ($\gamma_p = 0$), and other creep models are based explicitly or implicitly upon various combinations of the components of Eq. 1. The failure process, particularly time to failure, is either not considered (in most cases) or is assumed to take place in the later (secondary or tertiary) stages of creep. Moreover, often the ice strength is considered a special field of study loosely related to the creep of ice.

In the second approach initiated by this author, deformation and failure of ice are considered to be a unified process that takes place in all stages of creep. Consequently, the creep process is not considered in terms of real time (t) but in terms of normalized time $\bar{t}=t/t_m$, where t_m is time to failure (Fish, 1976, 1980; Morland and Spring, 1981; and others).

Time to failure plays an extremely important role within the framework of this approach. It is one of the most important parameters that define the service lifetime of a structure. Time to failure unites all three stages of creep and failure, but it also combines creep and fracture, microcrack formation kinetics, recrystallization, and other processes that take place during deformation and failure of ice (Fish, 1976).

The principal elements of the model, which was originally developed (Fish, 1976, 1980, 1983, 1987) to describe one-dimensional creep and failure of frozen soils and ice, and is expanded below for 3-dimensional deformation, can also be approximately expressed in terms of the components of Eq. 1 as a product of primary, secondary, and tertiary creep strains, i.e.:

$$\gamma^c = \gamma_e + \gamma_p \gamma_m \gamma_t \tag{2}$$

where γ_m = Constant = viscous shear strain at failure. Both primary and tertiary creep strains are functions of normalized time, i.e., $\gamma_p = \gamma_p(\bar{t})$ and $\gamma_t = \gamma_t(\bar{t})$. Secondary creep is considered to be a point (M) on the creep curve (Fig. 1) defining time to failure. The stress dependency of the strain rate that reaches a minimum at this point is described by a viscous flow equation that, as well as the time to failure function, includes a yield criterion of ice.

Thus the combined creep and yield model of ice under multiaxial stress developed in the following sections consists of four principal elements: a constitutive equation, a viscous flow equation and a yield criterion, all united by a time to failure function. One of the specific features of this model is that any of its elements can be replaced if it is found that other functions better represent the mechanical behavior of ice during creep.

CREEP MODEL

Triaxial Creep Tests (τ_i = Constant)

Constitutive equation

The entire deformation process, which includes primary, secondary, and tertiary short-term creep of homogeneous and isotropic ice under multiaxial stress at constant temperature, can be presented as a product of the flow equation and the nondimensional time function (Fish, 1980, 1983):

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KEY WORDS: Creep, failure, ice, model, strength, yield.