

# The Computation of Mean Drift Forces and Wave Run-Up by Higher-Order Boundary Element Method

Y.H. Liu, C.H. Kim\* and M.H. Kim\*

Department of Civil Engineering, Texas A & M University, College Station, Texas, USA

## ABSTRACT

A higher-order boundary element method (HOBEM) is used for the reliable computation of the first- and second-order wave loads on arbitrary three-dimensional bodies. The accuracy and efficiency of the present method are demonstrated through the convergence tests as well as comparison with analytic solutions and axisymmetric results. HOBEM is shown to be efficacious especially for the computation of body-surface velocities and velocity potential and its derivatives on the waterline and related integrals. For illustration second-order mean forces and wave run-up on vertical circular cylinders, hemispheres and the stationary ISSC tension leg platform (TLP) are presented.

## INTRODUCTION

During the past two decades, a number of numerical methods in solving water wave interaction with three-dimensional bodies have been proposed. Among them, the boundary integral equation method with the free-surface Green function has been studied most extensively due to its higher efficiency compared to the other numerical methods. Most of the previous studies on this subject are based on the constant panel method (CPM), which was originally developed by Hess & Smith (1964) and uses piece-wise constant potential on each plane panel. On the other hand, the numerical results based on a more elaborate higher-order boundary element method (i.e., high-order variation of the potential and geometry) are surprisingly rare (Tong, 1989; Chau, 1989). According to a recent study of Romate (1989) on the comparison of various boundary element methods, it is shown that the convergence of a constant panel method (CPM) in calculating first-order quantities on a cylinder with square cross section is much slower than the other higher-order boundary element methods.

Recently, a robust and versatile higher-order boundary element method (HOBEM) that allows arbitrary higher-order variation of the potential and geometries (e.g., quadratic variation of the potential on each quadratic isoparametric element) was developed by Liu et al. (1988, 1990). In this method, a fictitious potential is introduced in Green's formulation to be able to remove the singularity of the diagonal terms in the influence-coefficient matrix. Therefore, controversial problems such as the solid angle at the corners and edges are completely removed. In addition, a specially devised polar-coordinate transformation technique is used to integrate the Rankine singularity in a robust way.

Compared with conventional panel methods, the convergence of HOBEM is considerably improved and more accurate results can be obtained with fewer boundary elements, which will substantially reduce the CPU time. In addition, allowing higher-order variation of the potential, the HOBEM results are not sensitive to different discretizing schemes. For example, it is shown in Liu et

al. (1990) that the equivalent accuracy of Korsmeyer et al. (1988) with 4048 plane panels is obtained using only 152 quadratic elements in computing first-order wave loads on the ISSC TLP. Another salient feature of HOBEM is the capability of a direct interaction with most finite element programs for ensuing structural analyses.

The accuracy and efficiency of the first-order computation should be the firm basis for the subsequent second-order computation. It must be remembered that the errors created by the first-order module can be greatly amplified in the second-order computation. To illustrate the accuracy and efficiency of HOBEM, we computed second-order mean drift forces and wave run-up for vertical circular cylinders and spheres, and compared them with analytic or axisymmetric solutions (Kim and Yue, 1989). Particularly, the vertical mean forces on floating truncated cylinders, for which slow convergence is experienced by a direct pressure integration with the CPM (Zhao and Faltinsen, 1989), are computed using HOBEM, and a relatively rapid convergence is obtained. Finally mean drift forces and wave run-up for the stationary ISSC TLP are presented.

## MEAN DRIFT FORCES AND WAVE RUN-UP

We consider the second-order wave interaction with three-dimensional bodies. Cartesian coordinates with the (x,y)-plane in the quiescent free surface and z positive upward are chosen. Assuming ideal fluid and weak nonlinearities, we express the velocity potential  $\Phi$  as a perturbation series:

$$\Phi = \varepsilon \Phi^{(1)} + \varepsilon^2 \Phi^{(2)} + \dots \quad (1)$$

At each order, the velocity potentials can be decomposed into incident, diffraction, and radiation potentials;  $\Phi^{(i)} = \Phi^{(i)}_I + \Phi^{(i)}_D + \Phi^{(i)}_R$ ,  $i = 1, 2$ . In the presence of a regular wave of amplitude  $A$ , and frequency  $\omega$ , we have, at first order, the wave-frequency potential  $\phi^{(1)}$  whose amplitude is proportional to  $A$ , and, at second order, the time-independent ( $\bar{\Phi}^{(2)}$ ) and double-frequency ( $\phi^{(2)}$ ) potentials whose amplitudes are proportional to  $A^2$ . Similar expressions can be used for the forces  $\bar{F}$ , and free-surface elevation  $\zeta$ :

$$\begin{pmatrix} \Phi(\bar{x}, t) \\ \bar{F}(t) \\ \zeta(\bar{x}, t) \end{pmatrix} = \text{Re} \left\{ \varepsilon \begin{pmatrix} \phi^{(1)}(\bar{x}) \\ \bar{f}^{(1)} \\ \eta^{(1)}(\bar{x}) \end{pmatrix} e^{-i\omega t} + \varepsilon^2 \left( \begin{pmatrix} \bar{\phi}^{(2)}(\bar{x}) \\ \bar{f}^{(2)} \\ \bar{\eta}^{(2)}(\bar{x}) \end{pmatrix} + \begin{pmatrix} \phi^{(2)} \bar{e}_{c\bar{x}} \\ \bar{f}^{(2)} \\ \eta^{(2)}(\bar{x}) \end{pmatrix} e^{-i2\omega t} \right) \right\} \quad (2)$$

\*ISOPE Member.

Received March 1, 1991; revised manuscript received by the editors October 19, 1992. The original version (prior to the final revised manuscript) was presented at the First International Offshore and Polar Engineering Conference (ISOPE-91), Edinburgh, United Kingdom, August 11-16, 1991.

KEY WORDS: Higher-order boundary element method, drift forces, mean wave run-up, tension leg platform.