International Journal of Offshore and Polar Engineering
Vol. 3, No. 1, March 1993 (ISSN 1053-5381)
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# A Stochastic Roll Response Analysis of Ships in Irregular Waves

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## ABSTRACT

**The roll response of a ship to random beam seas is investigated in terms of the threshold crossing process. The nonwhite excitation process is modeled as an equivalent white-noise one, based on the assumption that the crossing properties of the response can be approximately replaced by the excitation with a white-noise process of a suitable intensity. Then the full nonlinear damping and nonlinear restoring are reinstated. The reinstated equation of motion with the equivalent white-noise intensity is solved using the average method to get the desired probability density function. The proposed scheme is tested extensively with various coefficients.**

## INTRODUCTION

The present study investigates a method of predicting the threshold crossing time by solving a nonlinear rolling equation of motion of a ship in irregular waves. Since the nonlinear nature of the rolling motion of a ship in waves is very complicated, a complete analytic solution of the problem has not been proposed so far.

Dunne (1985) has developed a new approximate method for dealing with nonlinear systems which are disturbed by excitation that can not be adequately classed as wideband, and modeled as white-noise. He combined the method of equivalent linearization and the FPK technique to obtain some useful results. The method has been applied to estimating threshold crossing rates. The theoretical results were tested with extensive simulation results. That is, their method is based on the assumption that the crossing properties of the response can be approximately replaced by the excitation with a white-noise process of suitable intensity. Thenthey reinstated the nonlinear restoring function from the equivalent linearized equation of motion.

The present study reinstates the full nonlinear damping and nonlinear restoring function with the equivalent white-noise intensity so that the nonlinearity in the damping can be adequately modeled. This white-noise-excited nonlinear equation of motion is solved by the average method to obtain the needed joint probability density function.

## METHODOLOGY

If the influence of all other degrees of freedom can be neglected, the equation of motion of a ship rolling in random beam waves can be written in the following form:

$$\ddot{\theta}+D(\dot{\theta})+R(\theta)=n(t) \tag{1}$$

where $\theta$ is roll angle, $D(\theta)$ represents a damping function, $R(\theta)$ represents a restoring function, and $n(t)$ is a Gaussian random process with zero mean and spectrum $S(\omega)$. The time scale is chosen so that the undamped natural roll frequency is unity. The damping and restoring function can be represented as:

$$D(\dot{\theta})=C_1\dot{\theta}+C_3\dot{\theta}^3 \tag{2}$$

$$R(\theta)=\theta+K_3\theta^3 \tag{3}$$

The present method is based on the assumption that the crossing properties of the response to an excitation can be approximated by replacing the excitation with a white-noise process of suitable intensity $J$, which was proposed by Dunne (1985).

First the system is linearized in a conventional way. The equivalent linearization technique applied to Eq. 1 replaces the system by an equivalent linear system:

$$\ddot{\theta}+C_{eq}\dot{\theta}+K_{eq}\theta=n(t) \tag{4}$$

The equivalent linear coefficients in Eq. 4 are determined by minimizing the mean square of the linearization error.

For the linearized system, the mean upcrossing times $\mu_{S(\omega)}(a)$ and $\mu_J(a)$ can be readily obtained. $\mu_{S(\omega)}(a)$ and $\mu_J(a)$ represent the mean upcrossing times which are calculated from nonwhite process and white noise, respectively. The nonwhite process describes the exciting moment due to wave excitation.

The value of $J$ is chosen to minimize the square error:

$$\int_0^{a_{\max}}\left[\mu_{S(w)}(a)-\mu_J(a)\right]^2 da \tag{5}$$

where $a_{\max}$ is a suitable high threshold. The newly determined $J$ is the best fit for the mean upcrossing time functions over the range of threshold of interest. If one reinstates the nonlinear damping and the nonlinear restoring, Eq. 1 can be rewitten as:

$$\ddot{\theta}+D(\dot{\theta})+R(\theta)=\sqrt{J}\,Z(t) \tag{6}$$

where $Z(t)$ represents the white-noise excitation. If we can get the probability density function of Eq. 6, then the expected values needed to calculate the equivalent linear coefficients in Eq. 4 can

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Received December 13, 1991: revised manuscript received by the editors August 12, 1992.
KEY WORDS: White noise, nonwhite noise, nonlinear roll equation, equivalent linearization, average method.