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Evolution of Long Nonlinear Gravity Waves on Shelves

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ABSTRACT

The problem of tsunami wave evolution is considered on the basis of a nonlinear dispersive model. The introduction of 3 small parameters characterizing nonlinearity, dispersion and inhomogeneity (slowly varying depth) makes it possible to obtain evolutionary differential equations generalizing the Korteweg-de Vries equation. The influence of bottom friction is also considered. The evolution of a solitary tsunami wave moving over the uneven bottom is investigated asymptotically and numerically. Formulae for the coefficients of amplification of wave height and wave steepness on shelves are proposed. The significant role of nonlinear- dispersive effects, and the effect of bottom friction on shelves is shown.

NOMENCLATURE

A : slowly varying parameter in a Gardner soliton
a : slowly varying parameter in a KdV soliton
B : slowly varying parameter in a Gardner soliton
b : slowly varying parameter in a solitary wave
D : depth
g : gravity acceleration
h : deviation of free surface
H : function, proportional to this deviation
K : coefficient of amplification
Q : soliton velocity
R : additional operator for KdV equation
t : time
u : terms of perturbation series for v
v : function, proportional to deviation of free surface
V : wave velocity
w : solution of conjugated equation
x : vertical coordinate
α : parameter of generalized inhomogeneity
β : parameter of inhomogeneity (characterizes bottom slope)
γ : wave steepness
Δ : efficient wave length
δ : dispersive parameter
ε : parameter of nonlinearity (ε_0 - initial value)
λ : wave length
μ : parameter of friction
σ : very slow horizontal coordinate
ξ : phase variable
ψ : function, proportional to velocity potential
θ : phase variable, depending on ξ and τ

INTRODUCTION

The evolution of surface gravity waves on shallow water is a serious scientific problem which has drawn the attention of scientists for more than one year. This problem is complex due to nonlinearities, and nonlinear effects are rather significant in many cases. In case of shallow water (long wave approximation) the bottom relief is a significant factor in the evolution. In practical cases of ocean wave studies, the possible bottom shapes are so different that there is no chance of neither their analytical description, nor for a computer investigation of all possible cases. Therefore the construction of asymptotic estimations, which are in accord with physical reality, at least for some class of problems, is a constructive approach.

This paper contains a brief derivation of evolutionary equations generalizing the variable coefficient Korteweg-de Vries equation (Kakutani, 1971; Johnson, 1973). The first section of this work is devoted to this question. The second part contains an asymptotic investigation of soliton evolution over a slowly varying uneven bottom. The formula for a law of wave height evolution was obtained in the main order in other works (Grimshaw, 1970; Ostrovskij and Pelinovskij, 1970; and others). The new aspects of the present work are the following: changes in the evolution law of the wave height due to the consideration of the next nonlinear term (the Gardner equation), the influence of dissipative processes on the parameters of the solitary wave, the second order terms in the formula for the wave velocity, the dynamics of the effective soliton length.

The main practical defect of the asymptotic theory is its limitation to a rather narrow class of problems restricted to small parameters. Therefore the following tasks are important:

(i) numerical descriptions of the applicability zones of asymptotic theory, and

(ii) investigations of the character of evolution outside these zones.

A series of numerical simulations was carried out. They are described below. The curves describing the dependence of the wave height on depth were obtained as the result of these simulations. These curves occupy the area between Green's law and the nonlinear asymptotic law. The form of these curves was the reason for the following idea: The dynamics of evolution are guided by inner (nonlinearity, Ursell parameter) and outer (slowly varying depth, coefficient of bottom friction) parameters. The introduction of a parameter of generalized inhomogeneity depending on nonlinearity, and changing during the process of wave propagation, permits estimation of its critical value, corresponding to the boundary point of the applicability of asymptotic theories. The procedure of joining two asymptotics at the critical value of this parameter enables us to obtain a semiempirical formula for

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