

Time Domain Models from Frequency Domain Descriptions: Application to Marine Dynamics

E. R. Jefferys

Conoco (UK) Ltd, Research and Development, London, United Kingdom

K.R. Goheen

Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa, Canada

ABSTRACT

Experimental or analytical modelling of physical systems, especially marine dynamic systems, often produces nonparametric frequency domain descriptions. Frequency domain models may be adequate for some purposes, such as acoustic or vibration analysis or feedback control system design by Bode or Nyquist methods. However, in many other circumstances, lumped parameter time-domain models are required. Typical applications include time-domain simulations on digital computers and design of modern or state-space controllers. Two innovative methods of fitting appropriate lumped parameter models are described here. The first fits $B(j\omega)$ to $G(j\omega)A(j\omega)$ in the square norm sense, a procedure which is usually reliable and computationally inexpensive. The second combines Inverse Fourier Transforms and System Identification to produce discrete-time difference equations optimised in the time domain. The methods are applied to two typical marine systems to illustrate their characteristics.

INTRODUCTION

Sometimes it is necessary to fit a lumped model to a complex frequency response which is known at a number of points. The latter description is a nonparametric model in that it cannot be characterized by a finite number of parameters; instead information is often presented in graphical form. It is usually necessary to do a parametric model fitting in order to carry out an efficient time-domain simulation of the system in question. Of course, it is possible to evaluate the time series response of the system to a known input by Fourier transformation of the input followed by multiplication in the frequency domain and inverse Fourier transformation, but this assumes an open loop system whose input is independent of its response. Alternatively, a convolution integral time-domain model can be derived by inverse Fourier transformation of the frequency response; the resulting time-domain model is relatively inefficient in simulations but can be very useful if an equivalent low-order differential equation model does not exist. Convolution calculations are faster than integration of the corresponding differential equation system if the ODE model is of high order.

The Convolution theorem states that an output $y(t)$ can be generated from an input $u(t)$ by a convolution with the system impulse response $h(t)$:

$$y(t) = \mathcal{L}^{-1}[G(s)u(s)] = \int_0^{\infty} u(t-t')h(t')dt' \quad (1)$$

$$\text{where } h(t) = \mathcal{L}^{-1}[G(s)] = \int_{-\infty}^{\infty} G(j\omega)\exp(j\omega t)d\omega$$

Typical systems of interest include the response of a distributed system at its boundary or the response integrated over the boundary if the system is more than one-dimensional; the fluid forces acting on a floating body caused by its motion are a good example

(Cummins, 1962; Van Oortmerssen, 1977; Korsmeyer et al., 1977). Instances may also be found in areas other than hydrodynamics. Mechanical properties of ships' machinery mounting seats may be modelled as complex transfer functions, derived from experimental results; the nonparametric frequency responses are measured at a representative point on the hull when a swept sine wave force input is applied at the machinery seat. The response of the complete system must be evaluated to ensure the ship's acoustic signature remains below acceptable levels; a lumped time-domain model could assist with noise reduction exercises by simplifying noise suppression calculations.

APPROACHES TO PROBLEM OF MODEL DERIVATION

The traditional method of obtaining lumped models from frequency response data uses the gain-phase information plotted on a Bode diagram (Franklin et al., 1986). Straight lines are fitted to the data, the breakpoints are estimated, which yields the poles and zeroes, and the damping ratios of the complex factors are estimated from the peak values of the amplitude spectrum. The phase information is used to determine whether the system is minimum phase. Bode (1940) showed that the phase of the output of a dynamic system must lag that of its input by at least a minimum amount — say, ϕ — which is defined by the shape of its amplitude spectrum; a nonminimum phase system would have phase $\phi \pm n360^\circ$, where n is an integer.

The difficulty with this technique occurs when information cannot be read accurately enough from the graphical construction; it is then very easy to identify poles and zeroes up to 50% away from their true values. In addition, this method becomes increasingly difficult to apply as the model order increases. Preferably, what is desired are methods that may be applied automatically and rationally to frequency responses of any complexity. Two such techniques are outlined here.

Again, we are interested in cases where the frequency domain data have been produced either from experiment or from a numerical method; at no stage are there necessarily time-domain data