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Vortex Formation in Ocean Currents

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ABSTRACT

The linear instability of surface currents is investigated, with the purpose of assessing the effect of the free surface on the form of the instability waves. The equations of motion and continuity are linearized around the time-average flow, assumed to have a self-similar form, and by considering wavy perturbations, the dispersion relation of the flow is obtained relating the frequency and wavenumber of the instability waves. The dispersion relation is solved for numerically to give the frequency as a function of the wavenumber. A range of unstable wavenumbers is found. The instability is of the convective type, in the sense that any localized perturbation will propagate outside any finite region. Consequently spatially growing perturbations are possible, which are determined numerically, using an iterative procedure. It is shown that even though the presence of the free surface has thus very little influence on the dispersion relation, which is almost identical to the one in unbounded fluid, it drastically alters the form of the instability wave by destroying the axisymmetry of the flow. As a result, the instability modes are not helical as in unbounded fluid, but are symmetric around the centerplane of the current, and cause fluid motion parallel to the ocean surface. Upper views of the instability modes are reminiscent of two-dimensional instability waves, even though the flow has been assumed equally sheared in all directions.

INTRODUCTION

Ocean currents develop large-scale meandering motions which lead to the pinching of vortices off the current. Spectacular pictures of meandering motions have been obtained in recent years from satellites (Legeckis, 1977; Legeckis et al., 1983) and have since attracted a lot of attention. Meandering motions have very long time and length scales, with time periods of the order of months, and wavelengths of the order of 1000 km (McPhaden et al., 1984; Miller et al., 1985; Philander et al., 1985). Meandering motions are very important for the stability of ocean circulation in regions where zonal currents are strongly sheared in depth and latitude. Moreover, vortices formed from the instability of ocean currents can cause significant climate variations, if they contain large masses of water of different temperature than their environment, and they can be dangerous for the safety of man-made structures, like offshore structures and mooring systems. The instability of ocean currents is a very complex process, influenced by such diverse factors as the rotation of the earth, and temperature and salinity variations in the region. A very considerable amount of work exists on the subject, where these factors are accounted for with the aid of some simplifying assumptions. (See McPhaden and Ripa, 1990, for a recent comprehensive review.)

In this paper, a relatively simple model is developed and investigated, with a very specific, and rather modest, purpose, namely to investigate the effect of the ocean surface on the development of shear flow instabilities in ocean currents. This effect is important because, as discussed in the next sections, it can account by itself for the shape of the observed meandering motions of ocean currents, and can be best demonstrated when the rest of the factors are not present. This statement is not intended to downplay the significance of the other factors, but rather to demonstrate the importance of the free surface in shaping the form of the instability modes. More specifically, ocean currents have very low Froude numbers, because of their low speed and large spatial extent. For instance, even for a strong current with velocities of the order of three knots, a spatial extent of 10 m only is enough to bring the Froude number to below 0.05. For such low Froude numbers, the dispersion relation is almost identical with the one occurring in unbounded fluid. Nonetheless, the observed instability patterns of ocean currents have a meandering form, and are therefore quite different from the instability patterns observed in axisymmetric jets, which have the form of rings and helices. This difference can often be attributed to the form of the velocity and density distribution in the current: If the velocity shear is much stronger along horizontal planes than along vertical planes, the instability motions will also be much stronger in the horizontal direction. In many cases, however, ocean currents are almost equally sheared in both planes, and they still develop meandering motions. In this paper it is shown that even in flows equally sheared in all directions, the vicinity of the free surface causes their instability modes to acquire a meandering form.

FORMULATION

We consider small perturbations around a parallel shear flow with a free surface (Fig. 1). The frame of reference has been chosen with the x-axis parallel to the main direction of the current, the z-axis parallel and opposite to the direction of gravity, and the y-axis perpendicular to the other two. The time-average velocity is denoted by $U(y,z)$. The equations of motion are the Euler equations for an incompressible fluid; at the free surface we have the kinematic and dynamic boundary conditions. The linearized equations of motion can be reduced to a single second-order partial differential equation in terms of the perturbation pressure p (Triantafyllou, 1990):

$$(kU-\omega)(\nabla^2 p - k^2 p) - 2k\nabla p \nabla U = 0 \qquad (1)$$

where $\nabla=(\partial/\partial y, \partial/\partial z)$. The boundary conditions at the ocean surface can be combined into the following condition (Triantafyllou, 1990):

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