International Journal of Offshore and Polar Engineering
Vol. 2, No. 2, June 1992 (ISSN 1053-5381)

Numerical Analysis of 2-D Nonlinear Cable Equations with Applications to Low-Tension Problems

Christopher T. Howell
Department of Ocean Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts
and
Department of Applied Ocean Physics and Engineering, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts, USA

ABSTRACT

Two efficient techniques are presented for analyzing cables under zero or low tension. First, an explicit finite-difference algorithm that is stable for zero-tension problems is presented. Secondly, an implicit finite-difference algorithm, which incorporates the effects of bending stiffness, is discussed. The importance of bending stiffness is shown for low-tension problems and the method proves stable, regardless of the cable tension magnitude. Therefore, the implicit method provides an efficient means with which to study low-tension cable problems in a more physically accurate way.

NOMENCLATURE

A	:	cable cross-sectional area
C_n	:	normal drag coefficient
C_t	:	tangential drag coefficient
Cd_n	:	$0.5 D\rho C_n$
Cd_t	:	$0.5\pi D\rho C_t$
D	:	cable diameter
E	:	Young's Modulus
I	:	moment of inertia
m	:	cable mass per unit length
m_l	:	cable mass + added mass per unit length
T	:	tension
u	:	tangential velocity $u_r = u - U_c \cos\phi$
U_c	:	horizontal current magnitude at time i+1
v	:	normal velocity $v_r = v + U_c \sin\phi$
w	:	cable weight in water per unit length
α	:	leading coefficient of dissipation term
ϕ	:	angle between cable and horizontal
ρ	:	fluid density

INTRODUCTION

The specific topics addressed within this investigation are the dynamics of low-tension cables and cables under zero initial tension. The term low-tension is used to refer to problems in which the cable static tension is significantly lower than the dynamic tension. This phenomenon can be restricted to finite regions, for example, near the free end of a towed cable, or it may occur along the entire cable length. In low-tension problems, the basic mechanisms that serve to propagate energy are altered and bending stiffness gains importance.

The cable-governing equations are nonlinear and strongly coupled. As a result, analytic solutions are unavailable, except in simplified cases. Therefore, in order to study the complete problem, numerical methods must be employed. A finite-difference approach was selected for this analysis. Existing finite-difference algorithms are limited in their application to low-tension problems for two reasons. First, they become singular if the tension vanishes anywhere along the cable length, a situation that is likely to occur in low-tension problems. Secondly, bending stiffness is neglected.

The limitations of existing algorithms created a need for novel approaches to the low-tension problem. Two alternative methods were developed to meet this need. The first method implements an explicit time integration finite-difference scheme that casts the cable tensions as the only unknowns in a matrix problem. Therefore, the onset of zero tension along the cable does not present a problem numerically. The second method developed incorporates the effects of bending stiffness in an implicit finite-difference formulation, similar to that of Ablow and Schechter (1983). An implicit scheme was selected in this case because the characteristics of the governing equations are altered by including bending stiffness.

EXPLICIT FORMULATION

The governing equations under investigation are the two-dimensional inextensible cable equations, expressed in local cable coordinates. These equations are derived by several authors including Bliek (1984) and thus are simply stated below without derivation.

$$m\left(\frac{\partial u}{\partial t} - v\frac{\partial \phi}{\partial t}\right) = \frac{\partial T}{\partial s} - w\sin(\phi) + Cd_t(U_c\cos(\phi) - u)\left|U_c\cos(\phi) - u\right| \quad (1)$$

$$m_l\frac{\partial v}{\partial t} + mu\frac{\partial \phi}{\partial t} + \rho A\frac{\partial U_c\sin(\phi)}{\partial t} = T\frac{\partial \phi}{\partial s} - w\cos(\phi) - Cd_n(U_c\sin(\phi) + v)\left|U_c\sin(\phi) + v\right| \quad (2)$$

These equations are based on a uniform current profile that may vary temporally. In addition to the governing equations, two compatibility relations are required. They are as follows:

Received May 25, 1991: revised manuscript received by the editors January 27, 1992. The original version (prior to the final revised manuscript) was presented at The First International Offshore and Polar Engineering Conference (ISOPE-91), Edinburgh, United Kingdom, August 11-16, 1991.
KEY WORDS: Cables, numerical methods, finite-differences, bending stiffness.