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Analysis of Nonlinear Response of an Articulated Tower

Oded Gottlieb, Solomon C.S. Yim* and Robert T. Hudspeth
Ocean Engineering Program, Oregon State University, Corvallis, Oregon, USA

ABSTRACT

A semi-analytical method is employed to investigate stability of the nonlinear response of an articulated tower. Local and global bifurcations determine the possible existence of complex nonlinear and chaotic motions which cannot be obtained through evaluation of an equivalent linearized system.

INTRODUCTION

Complex nonlinear and chaotic responses have been recently observed in various models of articulated towers and other compliant ocean systems (e.g., Thompson et al., 1984; Liaw, 1988). Similar behavior has been found in roll response of ships and semisubmersibles where the restoring moment was modeled by a quintic polynomial (Nayfeh and Khdeir, 1986; Witz et al., 1989). Articulated towers are surface piercing columns pinned to the sea floor which serve as mooring loading terminals for oil tankers. They are characterized by a nonlinear restoring moment and a nonlinear coupled hydrodynamic exciting moment. The restoring moment of the articulated tower is that of a forced plane pendulum and is generated by an internal excess buoyancy mechanism. The exciting moment includes a coupled wave-structure viscous drag component and a wave induced inertial moment. The drag component consists of parametric and quadratic damping, a bias and harmonic forcing.

The forced pendulum has been extensively investigated and complex nonlinear behavior, such as coexistence of attractors, symmetry breaking, period doubling and intermittency has been found experimentally, numerically and analytically (D'Humiers et al., 1982; Miles, 1988). Furthermore, global asymptotic criteria for the existence of chaotic response have been derived for the pendulum and for a Josephson junction circuit (Salam and Sastry, 1985) modeled as a forced and biased pendulum. However, unlike the unperturbed pendulum which has a pair of homoclinic orbits separating the domain of response into two disjoint parts of bounded and unbounded solutions, the articulated tower belongs to a family of oscillators which have a unique equilibrium position.

While weakly nonlinear systems have been studied extensively from both classical (Nayfeh and Mook, 1979) and modern approaches (Guckenheimer and Holmes, 1983), complex single equilibrium point systems are limited in their scope of analysis. Examples of these systems are the hardening Duffing equation analyzed by modified multiple scales and by the method of harmonic balance (Rahman and Burton, 1986; Szemplinska-Stupnicka, 1987) and the subharmonic motions of a wind loaded structure analyzed by the general method of averaging (Holmes, 1980). Stability analysis of system behavior results in local bifurcation maps defining regions of existence of the various nonlinear phenomena in parameter space. This analysis consists of perturbing the approximate solution and analyzing the resulting variational equation numerically by Floquet analysis or by analytically solving the equivalent Hill's variational equation. Both methods have been successfully employed on the hardening and softening Duffing equation (Szemplinska-Stupnicka, 1988; Nayfeh and Sanchez, 1989).

Extensive investigation of the articulated tower model has been performed for various configurations of slender and non-slender towers (e.g., Patel, 1989; Chakrabarti, 1987) in which the response was assumed small and the nonlinear moments were equivalently linearized by various methods. However, equivalent linearization eliminates the possibility of obtaining coexisting solutions and other nonlinear phenomena. Various configurations of articulated towers moored to floating structures have revealed the existence of subharmonic and chaotic response. Three models describing these motions are a bilinear oscillator identifying a stiffness discontinuity due to slackening mooring lines (Thompson et al., 1984), and two nonlinear oscillators where the restoring moment is characterized by cubic (Choi & Lou, 1991) and quartic (Fujino & Sagara, 1990) polynomials respectively. These models assumed small amplitude response and the restoring moment consisted of a linearized buoyancy component and the complementary nonlinear mooring stiffness function. The hydrodynamic drag moment was simplified to a quadratic damping function in the latter model and was linearized in the former models.

This paper describes a semi-analytic stability analysis performed on the nonlinear response of an unconstrained slender articulated tower. Consequently, the restoring moment consists of only a nonlinear buoyancy component. In order to model the nonlinear wave-structure coupling effect, the exact relative motion quadratic drag component is retained. Thus, the predicted local and global bifurcations determine the complex nonlinear behavior found numerically and can serve as a reference for identification of the generating mechanisms of instabilities in models with nonlinear mooring functions.

MODEL FORMULATION

The articulated tower considered (Fig. 1) is modeled as a single degree of freedom (θ: pitch), hydrodynamically damped and excited nonlinear oscillator. The equation of motion is modeled by a relative motion Morison equation with frequency independent coefficients (Sarpkaya and Issacson, 1983). This

*ISOPE Member.
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