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Three Dimensional Analysis of a Marine Riser with Large Displacements

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ABSTRACT

A formulation to analyze the large displacement problem of a marine riser with two unequal principal moments of inertia of cross sections in three-dimensional space is presented. The method involves the utilization of the equilibrium equations in the axial direction and the stationary condition of an energy functional. The coordinates X and Y, twisting angle F, and tension T are the four dependent variables, while the arc length s in the equilibrium state is the independent variable that is changed to depth Z in the numerical implementation. A finite element method was developed to solve the problem of a riser with equal principal cross sectional moments of inertia. An example is given.

NOMENCLATURE

B	: $B = B_1 \ell^2 + B_2 m^2$
B_1	: Bending stiffness to the principal axis one of a section, $B_1 = EI_1$
B_2	: Bending stiffness to the principal axis two of a section, $B_2 = EI_2$
$\hat{b}$	: Unit vector of the binormal of the central line $\hat{b}(\hat{b} = b_x\hat{I} + b_y\hat{J} + b_z$
C	: Torsion rigidity, $C = GJ$
E	: Young's modulus
F	: Angle between η and $\hat{n}$
$\bar{f}$	: Distributed load $\bar{f} = f_x\hat{I} + f_y\hat{J} + f_z\hat{K}$
G	: Shear modulus
H	: Total water depth
$\hat{I}$	: Unit vector of X axis
J	: Torsional rigidity factor of a cross sectional area
$\hat{J}$	: Unit vector of Y axis
K	: Curvature of the central curve
K_1	: ℓK
K_2	: mK
$\hat{K}$	: Unit vector of Z axis
ℓ	: $-\cos F$, or total arc length
m	: $\sin F$
$M_{tex}, M_{nex}, M_{bex}$	: Components of the external couple in the directions of tangent, normal and binormal of the central curve
$\hat{n}$	: Unit vector of the principal normal of the central curve, $\hat{n} = n_x\hat{I} + n_y\hat{J} + n_z\hat{K}$
Q_ζ	: $Q_\zeta = T$
$\bar{Q}$	: Internal force, $\bar{Q} = Q_x\hat{I} + Q_y\hat{J} + Q_z\hat{K}$
s	: Arc length of the central line in equilibrium
S	: Arc length of the central line in virtual state
T	: Effective tension
$\hat{t}$	: Unit vector of the tangent of the central line
W_r	: Weight of the riser per unit length
δW	: Virtual work done by external forces
δ	: Variational operator
ξ, η, ζ	: Coordinates of principal torsion-flexure system
τ	: Torsion
τ_1	: Torsion of central curve
$(\ )'$	: $= d(\ )/ds$

INTRODUCTION

In the literature, procedures to analyze a riser system experiencing large displacements in three dimensions were given by Bernitsas (1982), Chucheepsakul (1983), Felippa and Chung (1981), Garrett (1982), McNamara, O'Brien and Gilroy (1986), O'Brien, McNamara and Dunne (1988). A system with a layered flexible pipeline section was studied by McNamara and Harte (1989). Almost all of them assumed that the total arc length of the riser is a known quantity, while the top tension is an unknown. This paper presents a method of analysis for a riser system having a specified top tension and an unknown total arc length between seabed and the support at the slip joint as shown in Fig. 1. The magnitude of the top tension is governed by either the operation requirement, the lifting capacity or the strength of the riser material. In the formulation a riser having unequal cross sectional moments of inertia is considered, although they are equal in the given numerical example.

The hybrid method used previously for two-dimensional cases (Huang and Chucheepsakul, 1985) is extended to the present problem. Four dependent variables are used: the two horizontal coordinates of the centroidal curve, the rotation of a section, and the tension in the riser. To obtain these four unknowns, two equilibrium equations and two variational equations are used.

It is assumed that the riser material is linearly elastic, the unstrained riser is straight, and the shearing deformation, warping displacements and the Brazier effect on moment-curvature relationship are negligible.

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KEY WORDS: Marine riser, large displacement analysis, variational method, nonlinear analysis, finite element method, space curve.