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The Influence of Prebuckling Deformations and Stresses on the Buckling of the Spherical Shell

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ABSTRACT

From Von Karman's large deflection equation of the plate and by assuming that a plate has an initial deflection in the form of a spherical cap, the equilibrium equation of a spherical cap subjected to hydrostatic pressure is obtained, simplified and solved in the same way as an equilibrium equation of a beam on an elastic foundation subjected to axial and lateral loads. The influence of prebuckling deformations and stresses on the buckling of the spherical shell may be evaluated, and the relation between the buckling strength of the spherical shell and the column is obtained by analyzing the buckling problem of the beam on an elastic foundation. The formula presented for calculating the stability of the spherical shell gives a lower limit of buckling pressure and is in good agreement with test data recorded in the literature.

NOMENCLATURE

BCOEF : beam-column on elastic foundation
D : flexural rigidity of plate
E, E_s, E_t : elastic, secant and tangent modulus of material, respectively
h, R : thickness and radius of spherical cap, respectively
k : rigidity coefficient of BCOEF
l : length of BCOEF or longitudinal length of spherical cap
n_1 : stress coefficient of spherical cap in longitudinal direction
p : uniform pressure perpendicular to plane of plate or external uniform pressure on surface of spherical cap and shell
P_E : buckling pressure of spherical shell corresponding to modulus E
q : lateral uniform load acted on BCOEF
T : axial compressive load acted on BCOEF
u, v, w : components of displacement of spherical cap
w_e, w_a : initial displacement of plate perpendicular to plane of plate
$w_0, w_{x=0}$: uniform displacement and displacement at apex of spherical cap, respectively
α_i : ($i = 1, 2, 3, 4, 5$) arguments of BCOEF
β : parameter describing characteristics of BCOEF
γ : load parameter of BCOEF
η : critical pressure (or stress) coefficient of spherical cap and shell
θ : geometrical parameter of spherical cap
ψ : nondimensional stress
Φ : parameter describing buckling state of a structure

INTRODUCTION

Timoshenko (1936) summarized the classical small deflection theory for the elastic buckling of a complete sphere as first developed by Zolley in 1915. The theory assumes that the buckling occurs under the external uniform pressure:

$$P_E = \frac{2Eh^2}{\sqrt{3\left(1-v^2\right)}\,R^2} = 1.21E\left(h/R\right)^2 \tag{1}$$

where E : Young's modulus
v : Poisson's ratio
h : shell thickness
R : radius to the midsurface of the shell

However, the elastic buckling loads of roughly one-fourth of those predicted by Eq. 1 were observed in earlier tests recorded in the literature. Various investigators have attempted to explain this large discrepancy by introducing nonlinear, large deflection shell equations. Although a lower buckling load was found by von Káman and Tsien (1939) that is in fair agreement with the early experiments, subsequent researches, as mentioned by Zhou (1979), find the reason this lower buckling load is unsuitable for practical use. Because the results of theoretical research do not correlate with experiments, Krenzke and Kiernan (1965) recommended in the DTMB (David Taylor Model Basin) report the following empirical formula in the design of the spherical shell:

$$p_{cr} = 0.84\sqrt{E_s E_t}\left(h_a/R_{10}\right)^2 \tag{2}$$

$$\sigma_{cr} = \frac{p_{cr}\left(R_{10}\right)^2}{2R_1 h_a} \tag{3}$$

where E_s, E_t : secant and tangent modulus corresponding to σ_α, respectively
h_a : average thickness over a critical arc length L_c
R_1 : local radius to the midsurface of the shell over the critical arc length L_c
R_{10} : local radius to the outside surface of the shell over the critical arc length L_c,

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KEY WORDS: Spherical shell, hydrostatic pressure, buckling strength, beam-column on an elastic foundation, column.