

## Transient Surge Motion of a Moored Ship in Random Waves

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### INTRODUCTION

A ship in waves is subjected to second-order wave forces as well as to linear oscillatory ones. The second-order force contains slowly varying components, of which the characteristic frequency can be as low as the natural frequency of horizontal motions of moored ships. As a consequence, the slowly varying force can excite a moored ship to undergo unexpectedly large horizontal excursions, which may result in serious damages to its mooring system.

Many investigators have tackled this problem under respective interests. For publications up to 1982, readers are referred to the comprehensive review article of Ogilvie (1983). Agnon et al. (1988) rigorously examined shallow-water effects on the slow drift motion of a floating cylinder by using multiple-scale expansion techniques in both time and space. They clarified under which conditions the slow motion can be large. Choi and Kim (1989) applied the method to a slender ship in beam seas.

Our primary concern here is focused on the second-order slowly varying drift force on and the transient surge motion of a moored ship in random head seas. In this paper, we include only the second-order incident waves among second-order potentials. More attention has been paid on the role of the time-memory effect on the surge motion of a moored ship in head seas.

### SLOW DRIFT FORCES

Under the usual assumptions for potential flow, all physical quantities involved are perturbed asymptotically with respect to the smallness parameter defined by the slope of incident waves. The leading-order potential turns out to be the solution of a well-known linear boundary-value problem, for which several algorithms are available.

A consistent analysis yields the second-order force at the next order (Pinkster, 1976), which consists of five parts: The first term is due to the square of relative wave height; the second integral corresponds to the velocity squared, which is always negative; the third one is the cross-product of linear translations and the gradient of linear hydrodynamic pressure, whereas the fourth one reflects the change in normal vector due to linear rotations; the last term represents the contribution of the second-order potential. These are denoted by (I), (II), (III), (IV) and (V) by Choi and Choi (1990).

As the slow motion is mostly affected by the difference-frequency component of the drift force, we take only this part into consideration, which can be expressed in terms of quadratic transfer functions as below:

$$F = \sum_i \sum_j \left| \zeta_i^{(1)} \right| \left| \zeta_j^{(1)*} \right| P_{ij} \cos \left\{ -(\omega_i - \omega_j)t + \varepsilon_i - \varepsilon_j \right\} + \sum_i \sum_j \left| \zeta_i^{(1)} \right| \left| \zeta_j^{(1)*} \right| Q_{ij} \sin \left\{ -(\omega_i - \omega_j)t + \varepsilon_i - \varepsilon_j \right\} \quad (1)$$

where  $\zeta_i^{(1)}$  and  $\varepsilon_i$  denote the wave amplitude and phase angle of the  $i$ -th harmonics, and the superscript \* stands for complex conjugate. Hereby  $P_{ij}$  and  $Q_{ij}$  are the quadratic transfer functions corresponding to in-phase with and out of phase the incident wave, respectively.

It is straightforward to calculate the first four components of the slow drift force once the linear solution is available. It is recalled that the complete second-order potential is required for evaluating the last term correctly. Some efforts have been made in this direction (Molin, 1979; Kim and Yue, 1989). However, the solution is still incomplete, so that we consider only the second-order incident waves.

Pinkster suggested that the force due to second-order incident waves may be estimated approximately in the same procedure as for the linear case, if the effect of gravitational acceleration is so manipulated that the dispersion relation is satisfied.

$$H_{ij} = \frac{1}{2} \frac{A_{ij}(\omega_i - \omega_j)}{g} F_{ij}^{(1)*} \quad \omega_i > \omega_j \quad \text{with} \quad H_{ij} = P_{ij} + iQ_{ij} \quad (2)$$

where  $A_{ij}$  denotes the coefficient of the second-order incident wave potential derived by Bowers (1976), and  $F_{ij}^{(1)}$  denotes the amplitude function of equivalent linear wave-exciting force for unit amplitude waves.

### TRANSIENT SURGE MOTION

The surge motion of a moored ship can be described by the following integro-differential equation (Wehausen, 1971):

$$\{M + a(\infty)\} \ddot{X}(t) + \int_{-\infty}^t L(t - \tau) \ddot{X}(\tau) d\tau + CX(t) = F(t) \quad (3)$$

where  $a(\infty)$  is the added mass at frequency infinity and  $L(t)$  the time-memory function.

The time-memory function is associated with hydrodynamic forces in terms of either added mass or wave damping in the light of the Cramer-Kronig relation. We note that the values of added mass at frequencies near zero and infinity are more or less uncertain, while those of wave damping tend monotonically to zero. It is thus desirable to derive the time-memory function from the Fourier inversion of wave damping:

$$b/\omega = \int_0^{\infty} L(t) \sin \omega t dt \quad (4)$$

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Received March 20, 1990; revised manuscript received by the editors June 28, 1991. The original version (prior to the final revised manuscript) was presented at The First Pacific/Asia Offshore Mechanics Symposium (PACOMS-90), Seoul, Korea, June 24-28, 1990.

KEY WORDS: Moored ship, transient surge motion, random head sea, time memory effect.