International Journal of Offshore and Polar Engineering
Vol. 1, No. 3, September 1991 (ISSN 1053-5381)

Steady Response of Multi-Leg Moorings by Direct Integration

R.B. Chiou and J.W. Leonard
Department of Civil Engineering, Oregon State University, Corvallis, Oregon, USA

ABSTRACT

In the design and analysis of multi-leg mooring systems, it is significant, but difficult, to determine the tensions in the mooring legs. The analysis is complicated primarily because of the nonlinear behavior of the cables. The equilibrium equations of the moored body are indeterminate if the number of mooring legs is more than the number of unconstrained degrees of freedom being considered at the moored body. In the presence of spatially variable sub-surface currents, it is generally not appropriate to approximate the cable behavior by catenary equation, as the current-induced drag forces are both position- and orientation-dependent. A method based on direct spatial integration will be demonstrated for the nonlinear static analysis of three-dimensional multi-leg mooring system response to steady currents. This method is well-suited for use on microcomputers.

INTRODUCTION

A multi-leg mooring system, which is comprised of cables, anchors and the moored body, provides efficient restraint in all directions to the moored body by transmitting the forces on the cables and moored body to the anchors. Such a system has been employed in a wide variety of applications in the ocean (Nordell and Meggitt, 1981; Knapp, 1987). A few examples of multi-leg mooring systems include the systems used to restrain tension leg platforms and guyed towers for deepwater oil operations. A state-of-the-art review regarding the behavior of cables as mooring system components, the types and selection of cables, and the various classes of anchors and their applications was presented by Skop (1988).

In the design and analysis of multi-leg mooring systems, it is significant, but difficult, to determine the tensions in the mooring legs. The analysis is complicated primarily because of the nonlinear behavior of the cables (Leonard, 1988). The equilibrium equations of the moored body are statically indeterminate if the number of mooring legs is more than the number of unconstrained degrees of freedom being considered at the moored body. Due to this reason, some analyses of multi-leg mooring systems are based on the mathematical programming, i.e., optimization schemes (Jones and Nelson, 1982; Sekita, 1982; Wilson and Orgill, 1984). In the presence of spatially variable subsurface currents, it is generally not appropriate to approximate the cable behavior by catenary equation, as the current-induced drag forces are both position- and orientation-dependent. The paper will demonstrate a convergence acceleration method using direct spatial integration for the nonlinear static analysis of three-dimensional multi-leg mooring system response to steady currents. Examples are included to demonstrate the capability of the method.

The mathematical model of the NL-leg mooring system may be formulated as a ($NL + 1$)-point nonlinear boundary-value problem. The boundary-value problem, posed as a set of nonlinear differential equations, is first transformed into an iterative set of quasi-linearized boundary-value problems. Each quasi-linearized boundary-value problem is then further decomposed into a set of initial-value problems so that spatial integration may be performed along each mooring leg. The solutions to each of the quasi-linearized initial-value problems are recombined so as to always satisfy the boundary conditions during the iterations. The nonlinear boundary-value problem is then solved by successive iterations similar to the relaxation methods (Press et al., 1986). The algorithm, using the Newton-Raphson method of gradient scaling for nonlinear first-order differential equation, has been shown to be quite efficient (Leonard, 1979; Chiou, 1989). The advantage of this approach is that only a relatively small amount of variables need to be retained in computer memory. Thus, the solution for a large set of simultaneous algebraic equations, as generated by lumped parameter methods or finite element methods, can be avoided.

SOLUTION ALGORITHMS

Newton-Raphson Quasi-Linearization

Given a coupled set of nonlinear first-order differential equations, it is possible to develop a convergence acceleration procedure of successive iteration upon quasi-linear equations. In this section, the conversion of a nonlinear two-point boundary-problem into an iterative set of linear two-point boundary-value problems will be presented.

Assume a set of $2N$ nonlinear first-order differential equations:

$$\left\{\frac{dY_i}{dS}\right\} = \left\{f_i\left(S, Y_j\right)\right\} \qquad i,j = 1,2,\ldots,2N \tag{1}$$

with N nonlinear boundary conditions at boundary $S = 0$:

$$\left\{\bar{h}_k\left(\bar{Y}_j\right)\right\} = \{0\} \qquad k = 1,2,\ldots,N \tag{2a}$$

and N nonlinear boundary conditions at boundary $S = L_o$:

$$\left\{\bar{\bar{h}}_k\left(\bar{\bar{Y}}_j\right)\right\} = \{0\} \qquad k = 1,2,\ldots,N \tag{2a}$$

where S is the independent variable (e.g., cable arc length), $\{Y_i\}$ are the $2N$ dependent variables (e.g., tension components and location coordinates), $\{f_i(S, Y_j)\}$ are nonlinear functions of $\{Y_j\}$, and $\left\{\bar{h}_k\left(\bar{Y}_j\right)\right\}$ and $\left\{\bar{\bar{h}}_k\left(\bar{\bar{Y}}_j\right)\right\}$ are nonlinear functions of $\left\{\bar{Y}_j\right\}$ at $S = 0$

Received May 18, 1991: revised manuscript received by the editors July 24, 1991. The original version (prior to the final revised manuscript) was presented at The First European Offshore Mechanics Symposium (EUROMS-90), Trondheim, Norway, August 19–22, 1990.
KEY WORDS: Cable, mooring, hydrodynamics.